Crashworthiness and optimization of novel concave thin-walled tubes

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Abstract

In this work, a new type of energy-absorbing thin-walled tubes with concave angles is proposed by a unique structural design method to improve the crashworthiness performance of the traditional hexagonal thin-walled tube (TH). These concave tube structures (CTSs) are named CTS1, CTS2, and CTS3 respectively, while CTS1 is developed by a common design method. The crushing behaviors of the CTS3 are investigated by quasi-static compression experiments and numerical simulations. The crashworthiness and energy dissipation mechanism of all the tubes are investigated, and the results demonstrate that the CTS3 exhibits superior energy absorption capability than the other proposed tubes and TH with the same mass. Then, the mean crush resistance of the CTSs is predicted by theoretical analysis, and the influences of slenderness rate, boundary condition, and loading rate on the crushing responses of CTS3 are performed by numerical analysis. Besides, the comparative analysis of performances of the CTS3 and the typical concave tubes (TCTs) is carried out, and the results indicate that the CTS3 has the best energy absorption capacity among these tubes. In addition, the optimal structure parameters of CTS3 are explored to enhance the capacity of energy absorption further.

Keywords Crashworthiness; Thin-walled structures; Energy absorption; Concave tube; Optimization;

1. Introduction

Due to the advantages of its excellent energy dissipation capacity, low cost, and easy installation, thin-walled (TW) structure as an effective energy absorbing device has been extensively utilized in

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various engineering fields, such as automotive, train and aircraft[1–4]. Relevant studies have shown that TW metal structures subjected to impact loading dissipate the energy in the form of plastic deformation, local fracture, and tearing of the material[5]. Over the past few decades, the traditional TW tubes such as circular tubes[6–8], triangular tubes[9,10], square tubes[11–13], polygonal tubes[14,15], and multicellular tubes[16–19] have been widely studied by means of numerical simulation, experiment test and theoretical analysis [6,18–21]. The above pioneering works of literature mainly focus on regular convex polygon tubes. Although the studies have found that increasing the number of edges of a convex polygonal tube can improve the crashworthiness of the structure[22], the crushing force will reach a saturated state when the number of edges increases to more than 11 [23].

In order to break through the limitation of increasing the capacity of energy absorption of traditional TW tubes and improve the energy-absorbing characteristics of the traditional TW tubes, Duarte et al.[24][25][26][27] conducted many valuable experimental studies, and the results showed that empty TW tubes can achieve superior energy absorption capacity by filling lightweight cellular materials, such as closed-cell aluminium foams[28][29], advanced pore morphology (APM) foam elements[30], metallic hollow spheres (MHS)[31]. In addition, the deformation modes and failure mechanisms of these foam filled structures were also revealed, and carbons steel bar incorporated into foam materials to further improve the energy dissipation capacity of these structures[32]. Zhang et al.[33][34] studied the crashworthiness of metal thin-walled tubes with composite wrapping under static and dynamic loadings. They concluded the crashworthiness performance of the thin-walled tube could be enhanced significantly by adopting composite wrapping. However, these studies focus on improving the energy absorption capacity of the original structure by increasing the mass of the system. To improve the energy absorption capacity and efficiency of the empty tube without increasing the mass of the original design, some innovative energy-absorbing tubes with concave corners have been proposed. Mathematically, a concave polygon is defined as a polygon with at least one internal angle of 180 degrees[35]. Tang et al. [36] innovatively proposed a non-convex multi-corner TW tube based on the traditional square tube by introducing the concave angles. The results showed that the specific energy absorption of the tube is significantly improved. Fan et al.[37] proposed a 12-sided and
16-sided non-convex polygonal tube, and the study revealed that the energy dissipation of the structure can be effectively enhanced by increasing the number of concave corners within a certain range. In 2015, Reddy et al.[38] introduced concave corners in the cross-section of the tubes and proposed an optimal multi-corner tube with good energy dissipation capacity. In the same year, Abbasi et al.[39] designed a new polygonal TW tube with concave corners based on the traditional TW tube, which exhibited good energy dissipation performance. To fully understand the influence of different geometric parameters on the crashworthiness of concave polygonal tubes, Sun et al.[40] revealed the relationship between different design parameters and crashworthiness indices in 2017. In addition, the origami concave tube was innovatively designed by Li et al.[41], and it was found that the designed origami initiator can successfully induce the concave tube to produce progressive failure mode.

Up to now, the common way to introduce the concave angle is to change the direction of the corner element of the convex polygon, and this way is named evolution approach I (the common evolution method). Although this type of concave tube obtained through the evolution approach I has achieved good results to a certain extent, more research is needed to further enhance the energy absorption capacity of the structure by new evolution methods. In this work, the CTS2 and CTS3 designed by evolution approach II are developed to improve the specific energy absorption and weaken the instability of the crushing process. However, there is yet a systematic study on the energy absorption performance for the concave tubes that evolved through the evolution approach II in the existing literature. Therefore, it is necessary to explore the crashworthiness performance of concave tubes under different evolution approaches.

In the current work, Section 2 shows the different evolution processes of concave tubes and the introduction of crashworthiness indices. The specimen preparation and experimental setup for the CTS3, and the finite element (FE) model are shown in Section3. In Section 4, the results of numerical simulation for the CTS3 are validated through physical experiments. Then, the energy absorption characteristics of all the tubes are analyzed, and a theoretical analysis of the average crushing force of the CTSs is carried out based on existing theory. The influences of slenderness rate, boundary condition, and loading rate on the structure responses for CTS3 are investigated and discussed. Furthermore, the performance of the CTS3 is compared with the typical concave tubes (TCTs). Finally,
structural optimization is adopted to explore the best configurations of CTS3 in Section 5. The conclusions of this work are made in Section 6.

2. Description and energy absorption characterization of structure

2.1. Geometric description of structure

The whole evolution process from the convex polygon to the concave polygon is shown in Fig.1, taking the traditional hexagon as a benchmark. In order to obtain different types of concave polygons, the following two ways of concave angle evolution are presented in this work. One approach is to change the direction of the six convex corners of the hexagon, and the corner element is folded inward to form a non-convex polygon, as shown in Fig.1(b), which is characterized as the evolution approach I (the common evolution method). The other way is to bulge each edge of the hexagon to the center direction, as outlined in Figs.1 (c) and (d), which is defined as the evolution approach II. The geometric parameters of the structures are described in Fig.1, where $T$ and $W$ represent the thickness of the tube wall and the width of the tube, and the cross-section shape of each tube is equilateral. To provide better comparability between different tubes, all the tubes have the same cross-sectional area ($A=324 \text{mm}^2$) to maintain the identical mass. The wall thickness of all the tubes is $T=0.6\text{mm}$, and the height of all the TW tubes is $L=150\text{mm}$. The other parameters are $W_1=3W_2=4W_3=5W_4=90\text{mm}$.

2.2 Crashworthiness characterization

Some different crashworthiness indices have been proposed to characterize the performances of different structures. It usually includes five indices[42–45]:

The initial peak crushing force (PCF) is the maximum peak force that appears for the first time in the initial stage of crushing. The total energy absorption (EA) is defined as the total energy dissipation of the structure during the deformation process. It is calculated by the following expression:

$$EA = \int_0^d F(x) dx$$  \hspace{1cm} (1)

Here $F(x)$ represents the instantaneous crushing force, and $d$ represents the distance of axial crushing. Specific energy absorption (SEA) is the energy absorbed per unit of mass of the structure:
where $m$ is the total mass of the structure. Mean crushing force (MCF) is defined by,

$$\text{MCF} = \frac{\int_0^d F(x)dx}{d}$$

Crush force efficiency (CFE) is usually utilized to evaluate the consistency of the load and be expressed as follows,

$$\text{CFE} = \frac{\text{MCF}}{\text{PCF}} = \frac{\int_0^d F(x)dx}{d} \frac{1}{\text{PCF}}$$

### 3. Experimental setup and numerical model

#### 3.1 Experimental setup

The material used for the specimens is aluminum alloy, which belongs to the AlMgSi alloy series (0.8-1.2%Mg;0.4-0.6%Si;0.3%Cu;0.09%Mn;0.26%Cr;0.19%Zn;0.11%Ti). The thermal treatment status of the material is annealing. In order to obtain the real mechanical performance data of the material, the tensile test was carried out on a 100kN capacity Instron 3382 electronic universal testing machine (see Fig.2(a)). The true stress versus plastic strain curve of the material is plotted in Fig.2(b). The basic characteristic parameters of the material are: the density $\rho = 2700\text{kg/m}^3$, Poisson's ratio $\nu = 0.33$, Young's modulus $E = 65$ GPa, the initial yield strength $\sigma_y = 42.3$ MPa. As quasi-static loading conditions are simulated, the strain rate effect is not considered in the FE modeling.

The wire cut electric discharge machine (WEDM) melts metal materials by pulse electric spark generated with a movable continuous electrode wire, and the machining accuracy is $\pm 20\mu\text{m}$. The WEDM process method is used to cut aluminum alloy bars to obtain specimens required for the experiment[47,48]. The primary processing process is adopted as follows: drill a through-hole along the axis of the aluminum alloy bar, pass the electrode wire through the hole, and then perform the wire-cutting operation. The processed specimen with a height of 150mm is shown in Fig.3(a), and its cross-sectional size is shown in Fig.1(d). The same type of universal testing machine was used to perform the quasi-static compression test. The specimen is placed between two rigid platens, and the lower platen is fixed. To establish the quasi-static loading conditions and ensure no dynamic effect occurs during the experiment, the upper platen moves down at a speed of 0.5mm/s until it reaches the
specified crushing displacement \((d=110\text{mm})\). The entire crushing process for the specimen was recorded by a camera. The quasi-static compression test setup is shown in Fig.3(b).

### 3.2 Development of finite element model

In numerical simulation experiments, commercial software Abaqus/explicit is usually used as an effective solver. As illustrated in Fig.4(a), the model of CTS3 is located between two rigid plates, used as a representative FE model. The S4R elements are widely employed for modeling thin-walled structures due to their low cost of computation and high accuracy\[49\]. The tube model is meshed by the S4R node reduced integral shell element, and five integration points are along the thickness direction. The two rigid pressure plates are meshed using the four-node 3-D bilinear rigid quadrilateral element (R3D4). The general contact algorithm is adopted between the rigid plates and the specimen, as well as the self-contact of the specimen\[50,51\][52]. The Coulomb friction coefficient between the rigid plates and the specimen is set to 0.3\[43,53\]. The upper rigid plate only retains the movement in the axial direction, applying a downward displacement. All directions of the lower rigid plate are restricted. The true stress-strain data of the material in Fig.2 (b) is utilized in this numerical model. Fig.4(b) shows the SEA values of the specimens with different mesh sizes, indicating that the mesh size is equal to or smaller than 1mm that can achieve reliable results. Here, the characteristic size of the element is chosen as 0.8mm for the specimen. A trigger is used as a mechanism to lower the initial peak force and induce progressive failure modes in structures\[54\][33]. The triggers with a depth of 0.3mm are introduced below the top of the model of CTS3 according to the experimental observation. The layout of the trigger is shown in Fig.4. The kinetic energy should be much smaller than the internal energy to ensure that the simulation process is quasi-static\[55,56\].

### 4. Results and discussion

#### 4.1 Experiment and simulation results

**4.1.1 Validation of finite element model**

The deformation process of the CTS3 under the quasi-static crushing experiment is shown in Fig.5(a). It is observed that the initial folding position of the CTS3 occurs near the top and the whole
process basically presents a progressive crushing deformation mode. Fig.5(b) shows the numerical simulation results, and it is clearly found that the CTS3 has almost the same deformation modes as the experimental observations. It is very interesting to find from Figs.5 and 6 that the deformations of the adjacent corners are compatible during the crushing deformation process. The alternating folding deformation of the CTS3 is very coordinated. There is no conflict among the folding lobes, and the axial space is fully utilized. As a result, this makes the material deform more fully and improves the energy dissipation efficiency of the material plastic deformation. From the results of experiments and simulations, it can be observed that the entire crushing process is plastic folding deformation in a non-extensional mode.

The force-displacement curves of the tube are shown in Fig.7. On the whole, the force response curve obtained by numerical simulation matches well with the experimental result. It can be found that the tendency and fluctuation characteristics of the response curves are generally consistent. All in all, the numerical model can reproduce sufficiently accurate experimental observations regarding the deformation process and mechanical response of this structure under axial crushing. Therefore, the numerical model is credible enough and can be used in subsequent studies.

4.1.2 Characteristics of energy absorption

It should be mentioned that due to the large ratio of width to thickness of the TH specimen, non-compact deformation mode is likely to occur during the crushing process. To ensure the progressive deformation mode as much as possible, the bottom of the TH is clamped[57]. It can be seen from Fig.8(a) that the TH produces two buckling deformation layers. Fig.9(a) shows that the load fluctuation of the mechanical response curve of the TH is small. It is observed from Fig.8(b) that the successive lobes are stacked on the first fold layer, and the failure deformation mechanism of the CTS1 is the progressive buckling mode. The load fluctuation of the mechanical response curve is relatively stable, as shown in Fig.9(a). Meanwhile, the progressive failure mode of the CTS2 is presented in Fig.8(c). As the crushing distance increases, the folding lobes begin to accumulate alternately. It is also accompanied by the progressive deformation mode with a stable mechanical response curve. In addition, it can be seen from Fig.9(a) that the crushing force level of the CTS2 is
higher than the values of the TH and CTS1. Comparing the response curves shown in Fig.9(a), it is not difficult to find that the tendency of the force response curve of each specimen is generally similar.

To reveal the deformation mechanism of folding lobes for the tubes, the profiles of deformation modes for the tubes are shown in Fig.10. It is clearly seen that the number of lobes of the CTS3 is more than that of the others at the same crushing displacement. There are about 4 lobes in the TH, and the folding wavelength is relatively larger. It can be observed that the number of lobes is approximately 8, 10, and 12 from the CTS1 to CTS3. Since all samples have the same crushing distance, it is easy to understand that the half of wavelength ($\gamma$) of the structures from small to large is the CTS3, CTS2, CTS1, and TH, i.e., $\gamma_4<\gamma_3<\gamma_2<\gamma_1$. Generally speaking, the concave tubes form more lobes and smaller folding wavelengths than that of the TH. Therefore, more materials undergo plastic deformation and enhance the energy absorption capacity of the structure. It is noted that the CTS3 has the smallest folding wavelength and the largest number of lobes among all the tubes, which reveals that more energy is dissipated under the same crushing distance. The energy-displacement curves are presented in Fig.9(b), which can be obtained by the integration of force response curves. As respected, it is found from Fig.9(b) that the curve level of CTS3 is markedly higher than those of the other tubes. Hence, the CTS3 has the highest energy absorption capacity.

The important crashworthiness data are displayed in Table 1 and Fig.11. It is observed from the results that the peak crushing force values of the CTS1, CTS2, and CTS3 are around 2.78%, 18.01%, and 33.68% higher than that of TH, respectively. More importantly, the performance of the loading consistency of the CTSs is better than that of the traditional hexagonal tube, which implies that the process of energy dissipation is more stable. That is, the CTS3 is preferable compared with the other tubes in terms of crush force efficiency, as shown in Fig.11(b). In addition, compared with the TH, the MCF values of the CTS1, CTS2, and CTS3 are increased by 112.59% ,182.96%, and 257.41%, respectively. Moreover, a similar increase rate also applies to SEA due to the structures with the same mass. It is worth noting that the CTS3 has the highest value of SEA, MCF, and CFE among the tubes.

It is important to highlight that the CTSs are based on the traditional hexagon structure (TH) through two different evolution approaches; The CTS1 is derived from evolution approach I, while the
CTS2 and CTS3 are derived from evolution approach II. Based on the above analysis, we can draw the conclusion that the concave angle structures obtained by evolution approach II have obvious advantages in energy absorption compared with that obtained by the evolution approach I, which means that the concave structures obtained by the evolution approach II can dissipate more kinetic energy without increasing the mass of structures.

4.2. Theoretical analysis

It is very essential to assess the crashworthiness of TW structures by theoretical analysis, which can predict the theoretical MCF values of the structures under axial compression, to assist the design of impact-resistant structures. The simplified basic folding mechanism element was proposed by Chen et al. [18](as indicated in Figs.12 and 13(b)), and the total bending energy can be calculated as,

\[ W_{\text{bending}} = \sum_{i=1}^{n} \theta_i M_i \]  

(5)

where \( n \) and \( l \) represent the number of static plastic hinge lines and the total width of the flange plate respectively, and \( \theta_i \) denotes the rotation angle. \( M \) is the full plastic bending moment per unit length, which is determined by the following equation:

\[ M = \frac{\sigma_0 t^2}{4} \]  

(6)

where \( t \) is the thickness of the flange plate, and \( \sigma_0 \) is the equivalent plastic flow stress of the material used in the specimen, which can be determined by the expression[11]:

\[ \sigma_0 = \frac{\left( \int_{0}^{\varepsilon_0} \sigma d\varepsilon \right)}{\varepsilon_0} \]  

(7)

where \( \varepsilon_0 \) is the strain corresponding to \( \sigma_0 \) that represents the ultimate tensile strength of the material, and the value of \( \varepsilon_0 \) is 0.16 which can be obtained by the experiment, then \( \sigma_0 = 110.81\text{MPa} \).

It is observed from Section 4.1 that all the tubes exhibit non-extensional failure mode. For a representative angle element, the membrane energy is dissipated by developing extension and compression elements, as shown in Fig.13. Moreover, Zhang et al. [57] innovatively integrated the membrane deformation energy of angle element with the energy dissipation of the moving plastic hinge, which can be obtained by the following calculation formula,
\[ W_{\text{membrane}}^{\text{conner}} = \frac{4MH^2}{t} \cdot \frac{\tan \alpha}{0.164 \left( \frac{B/t}{t} \right)^{0.6} \left( \tan \alpha + 0.06/tan \alpha \right)} \]  

(8)

where the width of the flange plate is \( B \), and the angle between two adjacent flanges is \( 2\alpha \).

Due to the fact that the flange plate cannot be completely flattened under one folded wavelength, some researchers[11,18,58] proposed an effective crushing distance coefficient \( \kappa \), which is assigned to be 0.78 in the present study. Consequently, the external work is required to develop a folding wavelength, which is derived by the following equation.

\[ W_{\text{ext}} = 2HP_{m} \kappa \]  

(9)

where \( P_{m} \) is the MCF. The entire length of the lobe is defined as \( 2H \), which is the original height of the undeformed structure. Considering the system energy conservation, the sum of energy dissipation of bending and membrane deformation is equal to external work, which can be expressed as follows,

\[ W_{\text{ext}} = W_{\text{bending}} + W_{\text{membrane}} \]  

(10)

Thus, the theoretical average collapse forces of structures are determined by establishing the governing equation.

The CTS3 contains 12 concave-angle elements and 18 convex-angle elements, and these corner elements develop the same failure mechanism. Therefore, according to Eq. (10), the MCF of the CTS3 can be written as the following formula,

\[ 2HP_{m} \kappa = 2\pi ML + 12W_{\text{membrane}}^{c} + 18W_{\text{CV}}^{c} \]  

(11)

Furthermore, the following results can be obtained,

\[ \frac{P_{m}}{M} = \frac{1}{\kappa} \left[ \frac{\pi l}{H} + \frac{H^3}{t} \cdot \frac{\tan \alpha}{0.164 \left( \frac{B/t}{t} \right)^{0.6} \left( \tan \alpha + 0.06/tan \alpha \right)} \right] \]  

(12)

The value of \( \alpha \) is \( \pi/3 \), and \( l \) is the total length of the flange plate for the CTS3, namely \( l = 60B \). The half-wavelength \( H \) can be determined by the minimum balance condition,

\[ \frac{\delta P_{m}}{\delta H} = 0 \]  

(13)

Therefore, the mean crushing force under the quasi-static loading can be expressed as:

\[ P_{m} = \frac{129.98\sigma_{l}}{\kappa} B^{0.8}l^{1.8} \]  

(14)

In the same way, the theoretical MCF values of the CTS1, CTS2, and TH can be obtained,
respectively.

\[ P_m = \frac{77.98\sigma_0}{\kappa} B^{0.2} t^{1.8} \quad \text{CTS1} \]  

\[ P_m = \frac{102.16\sigma_0}{\kappa} B^{0.2} t^{1.8} \quad \text{CTS2} \]  

\[ P_m = \frac{26\sigma_0}{\kappa} B^{0.2} t^{1.8} \quad \text{TH} \]  

Through the above theoretical analysis, it is not difficult to infer that the MCF value of the CTS3 is the highest among all the tubes. Consequently, theoretical analysis can be used as a convenient calculation tool, which can help researchers quickly predict and evaluate the energy absorption performance of the structures. This is very significant to reduce the costs of numerical calculations and physical experiments. Besides, a very good agreement of MCF values between the results from theory, simulation, and experiment is observed, as outlined in Fig.14. It can be seen from Table 2 that the theoretical values of the MCF are close to the numerical and experimental results. Although the theoretical analysis slightly overestimates the MCF values of the CTSs on the whole, the theoretical results can still be used as a reference for preliminary evaluation of structural performance.

### 4.3 Parametric study

As mentioned previously, the CTS3 has the best capacity for energy absorption among all the tubes. Hence, the CTS3 is selected to study further. The energy dissipation performance of the TW tube is highly correlated with failure mode, boundary condition, and loading rate, so it is necessary to investigate the effects of these factors on the energy absorption of the CTS3. Here, the parametric study is carried out using the numerical model already validated in Section 4.1. In all cases, the trigger applied in the numerical model is the same as in Section 3. The default bottom boundary condition is unfixed.

#### 4.3.1 Collapse mode

In this section, the deformation mode of the CTS3 with different structural parameters is studied, which is important for understanding the energy absorption mechanism. Note that \( L/W \) and \( W/T \) are defined as global slenderness and local slenderness, respectively. Keep \( W=18 \), and \( T \) takes value 0.6,0.8,1.0,1.2,1.4 and 1.6 mm. Hence, four groups (i.e., group A to B) of CTS3 with different length \( L \)
was investigated.

A series of FE analyses have been conducted, and the results suggest that the collapse modes can be classified into four typical deformation modes, i.e., progressive buckling mode (PBM), mixed mode (MM), transition mode (TM), and unstable mode (UM), which are depicted in Fig.15. The specimen collapses in the pure inextensional mode (IM), which is defined as the desirable progressive buckling mode (see Fig.15(a)). As shown in Fig.15(b), the specimen forms the mixed deformation mode with the combination of the IM and extensional mode (EM), and the IM and the EM are marked by red and black solid line rectangles, respectively. The deformation mode that switches from the mixed mode or progressive buckling mode to global buckling mode is specified as transition mode (see Fig.15(c)). In addition, unstable mode, or global buckling mode is an undesirable deformation mode for energy dissipation, which is given in Fig.15(d).

The influences of the global and local slenderness on collapse mode are displayed in Fig.16. In the case of group A, it can be observed from Fig.16 that the specimens achieve the progressive buckling mode when $18 \leq W/T \leq 30$. As $W/T$ decreases, the specimens ($W/T=15$ and 12.86) develop the mixed deformation mode with a combination of the IM and EM. Then, the deformation pattern of the specimen with $W/T = 11.25$ exhibits the transition mode that switches from the mixed mode to the global buckling mode. As we know, the increase in length of the specimen leads to an unstable deformation mode. In group B ($L=180$mm), the unstable buckling mode is observed for the tube with $W/T=11.25$. It is an interesting phenomenon that the specimens with $18 \leq W/T \leq 30$ experience the progressive buckling mode for all groups except that the specimen ($L=240, W/T=18$) forms the mixed mode. Besides, when $L$ increases, the specimens with $11.25 \leq W/T \leq 15$ are more likely to develop undesirable deformation modes such as the transition mode and the global buckling mode.

To further illustrate the correlation between $L/W$, $W/T$, and the failure mode, the map of the deformation modes is shown in Fig.17. It can be found from the figure that the failure modes more or less display the progressive buckling mode when $18 \leq W/T \leq 30$. With the decrease of the local slenderness (when $W/T=15$), the collapse modes are switched from the progressive buckling mode to the mixed mode. It is also observed that the specimens are more prone to collapse in the transition mode ($W/T=12.86$) or unstable mode ($W/T=11.25$), respectively.
The crush response curves of all groups are plotted in Fig. 18. As shown in the figure, the force level gradually increases with the decrease of $W/T$ except the case of $W/T=11.25$ for the groups B, C and D. The force level of the cases dropped suddenly is attributed to its unstable failure mode. The tendency of the force response curve for $W/T = 12.86$ in all groups is prone to decline. Besides, the same tendency also is observed for the case of $W/T=11.25$ in group A. The phenomenon is caused by the deformation switching from the mixed mode to the unstable mode. Furthermore, the force response curves ($W/T=30$, $22.5$, and $18$) for all groups exhibit considerable regular oscillations in the plateau stage and then enter into the stage of densification. However, it is noted that the oscillation of the cases with $W/T=15$ in all groups is evident due to the generation of irregular folding lobes in the process of crush.

Meanwhile, the influences of $W/T$ on crashworthiness performance are presented in Fig. 19 and the FE results are summarized in Table 3. The key crashworthiness indices are calculated at 73% length of the specimens. From Figs.19(a) and (b), it is seen that the values of MCF and SEA increase gradually with the decrease of $W/T$ for all groups, respectively. However, one exception is the cases of $W/T=11.25$ for the groups C, B and D, and the values of the MCF and SEA are much lower than those of the cases with bigger $W/T$. For the case of $W/T=11.25$, the values of SEA, MCF and CFE for group A are higher than those of other groups. This is attributed to the fact that the specimen formed transition deformation mode in group A, while developing unstable deformation mode in the other groups. Therefore, the specimen in group A have higher energy absorption indicators. It is found from Fig. 19(c) that the PCF value in the four groups increases with decreasing of $W/T$, and the main reason is that the increase of wall thickness results in the initial stiffness of the structure is increased. Besides, the PCF value of the identical $W/T$ in different groups is almost the same, which is due to that the cross-sectional area of the cases in four groups is the same[50]. Furthermore, the CFE value slightly improves with the decrease in $W/T$ from 30 to 15 for all groups, as shown in Fig.19(d). It may be due to that the increase rate of the MCF value is higher than that of the PCF. However, the CFE value suddenly decreases when $W/T$ changes from 12.86 to 11.25, which is caused by the deformation mode switching to the UM.
4.3.2 Effect of boundary condition

In actual application scenarios of energy-absorbing structures, different installation approaches will result in different boundary conditions. Generally, one end of the energy-absorbing structure will be fixed on the engineering parts for service. Therefore, it is necessary to explore the influence of the fixed boundary conditions at the bottom of the structure on the energy absorption performance. Noted that keep the global slenderness invariant \((L/W=8.33, \ L=150)\), the crush performances of the structure under different \(W/T\) values are investigated. Furthermore, it is noted that the value of \(W/T\) varies with the parameter \(T\) while \(W\) remains a constant. More specifically, \(W=18\text{mm}\), \(T\) increases from 0.6 to 2.2mm with the step size of 0.4mm (including \(T=0.8\text{mm}\)).

The force-displacement curves and crushed morphologies for the specimens with or without bottom clamped boundary by numerical simulation are presented in Fig.20. As shown in Fig.20 (a), when \(18\leq W/T\leq 30\), the deformation modes of the specimens under different boundary conditions are almost the same, which displays the pure inextensional mode. In general, the force response curves of every specimen under different boundary conditions are nearly overlapped, and there is a slight difference in the range of the densification of curves. Consequently, the bottom fixed boundary condition has almost no influence on the structural collapse response for \(18\leq W/T\leq 30\).

As \(W/T\) is 12.86, 10 and 8.18, the crush response curves of these cases under different boundary conditions are presented in Figs.20(b), (c), and (d), respectively. As shown in Figs.20(b) and (c), the cases of \(W/T=12.86\) and 10 without bottom fixed collapse in the MM and TM, respectively, and the clamped cases achieve more regular lobes than those of the cases without bottom clamped. Both of the unconstrained cases generate some lobes at the initial stage and then change to undesirable failure mode. The crush response curves are almost identical in the early stage (\(W/T=12.86\) and 10 corresponding to \(d=60\) and 40mm, respectively), while the difference of the curves is formed in the latter stage, which is due to the specimen with the fixed boundary condition that results in more regular lobes. Thus, this leads to the force level being improved. When \(W/T=8.18\), the specimen without bottom constrain deforms in the UM, while the specimen with bottom fixed boundary condition collapses in the TM, as shown in Fig.20(d). This implies that more materials of the specimen
undergo plastic deformation, and more energy is dissipated. Accordingly, the force response curve of the specimen with the clamped boundary is much higher than that of the unclamped case.

The energy absorption indices of all the cases are measured at the compression displacement of \( d=110 \text{mm} \), which is listed in Table 4 and shown in Fig.21. According to Figs.21(a) and (b), in general, as \( W/T \) increases (18\( \leq W/T \leq 30 \)), the MCF and SEA values show a very similar trend for both boundary conditions. For CTS3 with bottom unclamped boundary, the values of MCF and SEA increase steadily with the decrease of \( W/T \) except for the case of \( W/T=8.18 \). From the cases of \( W/T=10 \) to 8.18 without clamped boundary condition, the values of MCF and SEA decrease abruptly, which is mainly caused by the switch of deformation mode from the MM to UM. When 8.81\( \leq W/T \leq 12.86 \), the values of MCF and SEA of CTS3 under the clamped boundary condition are higher than those of the unclamped cases. Particularly, for the case of \( W/T=8.18 \), the MCF and SEA values are significantly improved compared with the unclamped case. However, the SEA value does not increase linearly and drops slightly as \( W/T=8.13 \), which is due to the increase rate of structural mass higher than that of energy absorption. As shown in Fig.21(c), it is interesting to note that the bottom fixed boundary almost has little influence on the PCF values of all the cases. Fig.21(d) illustrates that the CFE value increases with the decrease of \( W/T \) (when 10\( \leq W/T \leq 30 \)) and then decreases (when \( W/T=8.18 \)) for both of the boundary conditions. Overall, the values of CFE for the cases with the clamped boundaries are improved compared with the unclamped cases.

### 4.3.3 Effect of loading rate

In order to fully understand the dynamic response of CTS3 under different load velocities \((V)\), the structural responses are investigated for load velocities of 10m/s, 20m/s, 30m/s and 40m/s. It is noted that the strain effect of aluminum alloy material is slightly sensitive\([59][50]\), and the strain effect is not included in numerical simulation. The crushing responses and crashworthiness performances of the CTS3 \((\text{L}=150 \text{mm}, \text{W}=18 \text{mm}, \text{T}=0.6 \text{mm})\) under different crushing rates are depicted in Fig.22. The key crashworthiness indices are given in Table 5, which are calculated at \( d=110 \text{mm} \).

As shown in Fig.22, the CTS3 under different loading rates exhibits the same deformation patterns\((d=110 \text{mm})\), collapsing in the inextensional deformation mode. Meanwhile, the corresponding
final profiles of failure modes \((d=110\,\text{mm})\) are also illustrated in the figure. It is observed that under different loading speeds, the number of lobes of the specimen is almost the same, about 12 lobes. However, it was also found that the material at the bottom of the specimen with a loading speed of 10 \(\text{m/s}\) was folded and deformed, which indicated almost no residual material and space for plastic deformation to absorb energy in the specimen. For the case of higher velocity, there is still a small distance at the bottom of the specimens that remains undeformed, and the length of uncompressed distance \(\lambda\) slightly increases with the increase of velocity \((\text{i.e., } \lambda_4 > \lambda_3 > \lambda_2 > \lambda_1)\). Therefore, the increase of the load speed has a positive effect on the energy absorption capacity of the structure.

As seen from Fig. 23, the crushing force level of the CTS3 increases steadily with the increase of crushing speed. There is almost no difference in the tendency of the crushing response curves under different loading velocities. All the cases experienced three typical stages, namely, the initial peak force stage, the crushing load stability, and the densification stage. According to Fig. 23 and Table 5, it is found that the values of PCF and MCF are affected by the load speeds. As the load speed increases from 10 to 40\(\text{m/s}\), the PCF and MCF values almost increase by 48.41\% and 34.24\%, respectively, which is attributed to the inertia effect under dynamic crushing velocity. Similarly, the SEA value increases from 10.53 \(\text{kJ/kg}\) to 14.14 \(\text{kJ/kg}\) when the speed increases from 10 to 40 \(\text{m/s}\). Therefore, the results reveal that the SEA, PCF, and MCF values increase linearly with loading velocity. In addition, it is noticed that the CFE value decreases obviously when the velocity changes from 10 to 30\(\text{m/s}\). However, the CFE value hardly changes as the velocity varies from 30 to 40\(\text{m/s}\).

As aforementioned in Section 4.3, the CST3 with different \(W/T\) collapses in different deformation modes at quasi-static compression. Hence, we attempt to investigate the influence of dynamic load on the deformation modes. The crushing velocity of 40\(\text{m/s}\) is chosen for dynamic analysis. The crushing response curves and collapse modes of the CTS3 with various \(W/T\) under dynamic and static loadings are presented in Fig. 25. Note that the \(W\) keeps constant \((W=18\,\text{mm})\) and \(T\) changes from 0.6 to 2.2 \(\text{mm}\), with the interval 0.4 \(\text{mm}\). In addition, \(T=0.8\,\text{mm}\) is also employed.

As the cases \((W/T=30, 22.5, \text{and } 18)\) still exhibit inextensical mode when subjected to dynamic and static loadings, thus, the failure modes are not displayed here for the purpose of space saving. Nevertheless, the crushing force curves of CTS3\((W/T=8.18, 10, \text{and } 12.86)\) under dynamic loading are
much higher than those under quasi-static loading, as depicted in Fig.25. This phenomenon is mainly due to the switch of deformation mode and inertia effect of CTS3 at dynamic loading. Besides, the values of PCF for quasi-static and dynamic cases are marked with blue and green circles respectively, and it is observed that the occurrence of PCF for dynamic loading is delayed compared with the quasi-static case.

From Fig.25, it is also found that the CTS3 develops stable failure mode under the dynamic loading, which exhibits more regular lobes in comparison with the quasi-static loading cases. It is worth noting that the deformation mode changes from the UM to mixed failure mode for $W/T = 8.18$, which is beneficial for energy absorption. Besides, the diamond mode occurs in the structure, as presented in Fig.25(c), which results in the lower force curve at the range of compression displacement $85 \leq d \leq 122$mm. In general, the dynamic loading can promote the structure to create a more regular failure mode, which leads to more materials undergoing plastic deformation.

The crashworthiness indices are shown in Fig.26, and it is found that the impact velocity has a positive effect on the MCF and SEA values of the specimens. Especially, the values of SEA and MCF for $W/T = 8.18$ are increased by 144.13% and 145.56%, respectively. There is a general increase in the PCF values with various $W/T$ for impact loading, and the highest increase percentage is 18% for $W/T = 18$. The values of CFE for various $W/T$ are higher than those of the cases under quasi-static loading, and the CFE values are increased by 6.15%-122.86% .

4.4 Comparison of crashworthiness between CTS3 and typical concave tubes

According to the analysis in Section 4.1, it can be clearly seen that the CTS3 has the best energy dissipation characteristics among those tubes. To evaluate the performance merits of the CTS3, the comparative analysis of energy absorption performance of the CTS3 and the typical concave tubes (TCTs) in the existing literature [36][37][60] is performed. The configurations of the TCTs are presented in Table 4. We conducted intensive numerical simulations for these structures. To make a fair comparison, all the numerical models are inputted the material data from Fig.2(b). It is worth mentioning that the TCTs and CTS3 have the same cross-sectional area($S=324 \text{mm}^2$), the thickness of the wall ($T=0.6$) and the height ($L=150$mm), thus maintaining the same mass. The crashworthiness indices of all the structures are calculated at the crushing distance of 110mm. It can be found from
Fig. 2(a) that the mechanical response curve level of the CTS3 is significantly higher than those of the TCTs. Thus, the CTS3 has the highest crush resistance. There is no doubt that the CTS3 has the highest energy absorption capacity among these structures, as shown in Fig. 2(b).

The crashworthiness indices and collapse modes of all the structures are listed in Table 6. The numerical results show that all the specimens exhibit a similar progressive failure mode. From Fig. 28(a), the SEA value of the CTS3 is much higher than those of the TCTs. Specifically, the highest and lowest increase rates of SEA are 120.3% and 26%, respectively. Due to all the structures having the same mass, a similar increase rate can be found for the crashworthiness indices EA and MCF. Furthermore, the CFE is usually used as a critical index to evaluate the crashworthiness performance of the tubes. As shown in Fig. 28(b), we can notice that the CFE value of the CTS3 is higher than that of the TCTs, which indicates the load uniformity of the CTS3 is better than that of the TCTs. The CTS3 exhibits obvious advantages in comprehensive energy absorption performance among these structures, and it has high potential as an energy absorption device.

5. Structural optimization

It can be demonstrated from the aforementioned analysis that the comprehensive energy absorption characteristics of the CTS3 are better than those of the other tubes. As a TW energy-absorbing structure, it is expected that the CTS3 has higher SEA and keeps PCF as low as possible. Therefore, PCF and SEA are taken as optimization objectives for the CTS3, and the following mathematical expression can describe the optimization problem,

\[
\begin{align*}
\text{Min } & \{ PCF(W, T), -SEA(W, T) \} \\
\text{s.t. } & 16 \leq W \leq 28 \\
& 0.5 \leq T \leq 1.3
\end{align*}
\]

(18)

Response surface method (RSM) is a fast and efficient approximation method, and it has been widely used in structural impact analysis [61–63]. Thus, the approximate models of PCF and SEA are constructed by using RSM. Sufficient sample points for constructing an approximate model were obtained through experimental design. Based on a full factorial experimental design strategy of 2 factors and 5 levels, 25 samples are taken, and the sample points are calculated by FE simulation. The
corresponding results are obtained in Table 7, as presented in Appendix A.

The response surface models of the PCF and SEA are established, as presented as follows.

\[
PCF = f(W, T) = -19.25 + 3.301W - 19.72T - 0.1599W^2 + 1.072WT + 53.23T^2
+ 0.003049W^3 - 0.06234W^2T + 1.995WT^2 - 23.23T^3
\]

(19)

\[
SEA = f(W, T) = 9.391 - 0.7434W + 26.54T + 0.004783W^2 - 0.3868WT - 1.17T^2
+ 0.0007469W^3 - 0.0419W^2T + 1.005WT^2 - 8.438T^3
\]

(20)

The accurateness of the response surface model is checked by the coefficients of multiple determination \(R^2\) and root mean square error \(\text{RMSE}\), given by[64],

\[
R^2 = 1 - \frac{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2}
\]

(21)

\[
\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}{N - P - 1}}
\]

(22)

where \(y_i\) is the response value calculated by FE method, \(\bar{y}\) is the mean value of \(y_i\), \(P\) is the number of non-constant terms in the response surface, and \(N\) is the number of sample points.

Generally speaking, the closer \(R^2\) is to 1, the smaller the \(\text{RMSE}\) is, and the more accurate the response surface model is. The values of \(R^2\) and \(\text{RMSE}\) for the fitted function of PCF are 0.99 and 0.27, respectively. For the fitted function of SEA, \(R^2 = 0.99\) and \(\text{RMSE} = 0.38\). Therefore, the established response surface model is sufficiently reliable. Figs.29 and 30 are the response surface function diagrams and contour maps of the PCF and SEA, respectively. From Fig.29(a), it can be seen that the PCF value increases with the increase of both \(T\) and \(W\). In particular, the effect of \(T\) on PCF is significantly greater than that of \(W\) (see Fig.30(a)). Fig.29(b) exhibits that when increasing \(T\), the SEA value is increased. However, \(W\) is increased, the SEA value is decreased. It is observed from Fig.30(b) that \(T\) has a remarkable influence on the SEA, whereas \(W\) has a minor influence on SEA.

Furthermore, the NSGA-II algorithm is used to optimize the two objectives of the structure in order to obtain the Pareto solution set[65,66]. Fig.31 shows the Pareto frontier obtained after optimization. From the figure, it can be clearly found that the optimization objectives of the SEA and
PCF are a pair of contradictory indicators. Nevertheless, in theory, any solution on the Pareto frontier can be considered as the optimal solution. The obtained Pareto frontiers are available for engineers to make decisions based on their actual demand. For example, when only the SEA is regarded as a single objective, the optimum point A for maximal energy absorption capacity is obtained. Compared to the original CTS3, the SEA value is increased by 117.1%. Conversely, if only the minimum impact acceleration is considered, the ideal point B is obtained, in which the optimized tube with minimum PCF, whereby the PCF value decreases by 31%. To balance the SEA and PCF indices, a compromised point, namely “Knee point”, is obtained by using the minimum distance selection method (TMDSM), which can be formulated mathematically as below[67–69]:

\[
\min D = \sqrt{\frac{1}{i} \sum_{c=1}^{i} \left( \frac{f_{cr} - \min(f_{r}(x))}{\max(f_{r}(x)) - \min(f_{r}(x))} \right)^2}
\] (23)

where \( f_{cr} \) is the \( r \)th objective function value in the optimum point \( c \), \( D \) represents the distance from the “Utopia point” to a point on the Pareto curve, and \( i \) is the number of objective functions. Compared to the original tube, the SEA value increases by 67.15%. However, the PCF value is higher than that of the original tube at the Knee point.

Finally, the comparison of numerical results with optimization solutions is shown in Table 8. It can be found that the maximum errors of PCF and SEA values between the optimization results and numerical results are 0.01% and 0.029% respectively, which signifies that the optimization results are adequately accurate.

6. Conclusions

The main contribution of this work is to enhance the energy dissipation capacity of the traditional hexagon thin-walled tube (TH) by a novel structural design method. The CTS1, as well as CTS2 and CTS3, are proposed by a common method and a novel structural design method respectively. The FE model of the CTS3 is first verified by the experimental results. Then, the performance of the CTSs has been systematically investigated by numerical approach and theoretical analysis. The major conclusions are summarized as follows:
(1) The results have demonstrated that CTSs exhibit superior energy absorbing performance compared with the traditional hexagon tube with the same mass. The number of fold lobes of CTS3 is more than those of the other tubes under the same crushing displacement. Besides, the SEA values of the CTS2 and CTS3 are about 2.83 and 3.57 times that of the TH, respectively. Especially, the SEA values of the CTS3 and CTS2 are about 68% and 33% higher than that of CTS1, respectively. Thus, the concave angle structures obtained by the novel structural design method have obvious advantages in energy absorption compared with that obtained by the common method.

(2) It is found that the collapse mode and performance of the CTS3 are dependent on the global and local slenderness. The classification of collapse modes for the CTS3 under different global and local slenderness are defined into four deformation modes (i.e., PBM, MM, TM, UM). Generally, the values of MCF, SEA and CFE increase gradually with the decrease of $W/T$ (when $12.86 \leq W/T \leq 30$ and $10 \leq L/W \leq 13.33$) in the vast majority of cases, while these crashworthiness indices suddenly decrease when $W/T = 11.25$. However, the PCF value is all increased with the decrease of $W/T$. In addition, the bottom clamped boundary condition can enhance the energy absorption capacity of CTS3 (when $8.81 \leq W/T \leq 12.86$), and has little influence on the performance when the structure collapses in the progressive buckling mode under the unclamped boundary condition.

(3) It is revealed that the impact velocity significantly influences the crashworthiness performance of CTS3, and the values of MCF, SEA, and PCF increase with the increase of load velocity. When $8.81 \leq W/T \leq 12.86$, the dynamic loading positively affects the deformation mode, which can impel the structure to develop more regular folding lobes. It is found that the CTS3 exhibits superior energy absorption capacity compared to the TCTs with the same cross-section area and thickness. Besides, it is also discovered that the effect of $T$ on PCF and SEA is significantly greater than that of $W$.

(4) Structure optimization is carried out to improve the energy absorption performance of CTS3 further. The optimization results present a set of optimum design points for engineers to get CTS3 with excellent crashworthiness performance. Moreover, the proposed CTS3 is very promising to be used as a potential energy absorber.
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Appendix

Table 7 Sampling points and corresponding response results.

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<th>NO.</th>
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<th>T(mm)</th>
<th>SEA (kJ/kg)</th>
<th>PCF (kN)</th>
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Figure captions

Fig.1. A sketch of different thin-walled tubes: (a) Traditional hexagonal thin-walled tube (TH), (b) Concave tube structure 1 (CTS1), (c) Concave tube structure 2 (CTS2), and (d) Concave tube structure 3 (CTS3).

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Fig. 16. Deformation modes of CTS3 with different $L/W$ and $W/T$ under quasi-static loading.
Fig. 17. The map of deformation modes of CTS3.
Fig. 18. The numerical results for force response curves of the CTS3 for (a) group A, (b) group B, (c) group C, and (d) group D.
Fig. 19. Effect of $W/T$ on crashworthiness performances for CTS3: (a) MCF, (b) SEA, (c) PCF, and CFE.
Fig. 20. The effect of boundary conditions on the deformation modes and force response curves of CTS3 with different $W/T$. 
Fig. 21. Column charts on crashworthiness indices of CTS3 with various W/T under different boundary conditions.
Fig. 22. Failure modes and folding lobes of the CTS3 under different load speeds by numerical simulation.
Fig. 23. The numerical results for force response curves of the CTS3 under different load speeds.
Fig. 24. The numerical results for force response curves under different loadings and collapse modes of CTS3 with various \( W/T \) under dynamic loadings (Note-MM: Mixed mode; TM: Transition mode; UM: Unstable mode.)
The numerical results for plots on crashworthiness indices of CTS3 with various $W/T$ under quasi-static and dynamic loadings: (a) MCF and PCF, (b) SEA and CFE.

Fig. 25.
Fig. 26. Crushing response for specimens CTS3 to TCT5 by numerical simulation: (a) Force-displacement curves and (b) Energy absorption versus displacement curves of the structures.
Fig. 27. Comparison of the numerical results for energy absorption indices of the different tubes: (a) SEA and (b) CFE.
**Fig. 28.** The response surface graphs of (a)PCF and (b)SEA.
Fig. 29. The contour maps of (a) PCF and (b) SEA.
Fig. 30. Pareto front of the CST3 structure.
**Table 1** Energy absorption indices of the different thin-walled tubes.

<table>
<thead>
<tr>
<th>Tube</th>
<th>PCF Inc (kN)</th>
<th>MCF Inc (kN)</th>
<th>SEA Inc (kJ/kg)</th>
<th>EA Inc (J)</th>
<th>CFE Inc value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH</td>
<td>15.44 0</td>
<td>3.22 0</td>
<td>2.70 0</td>
<td>353.96 0</td>
<td>0.21 0</td>
</tr>
<tr>
<td>CTS1</td>
<td>15.87 2.78</td>
<td>6.85 112.73</td>
<td>5.74 112.59</td>
<td>753.42 112.85</td>
<td>0.43 104.76</td>
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<tr>
<td>CTS2</td>
<td>18.22 18.01</td>
<td>9.11 182.92</td>
<td>7.64 182.96</td>
<td>1002.2 183.14</td>
<td>0.5 138.1</td>
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<tr>
<td>CTS3</td>
<td>20.64 33.68</td>
<td>11.51 257.45</td>
<td>9.65 257.41</td>
<td>1266.35 257.77</td>
<td>0.56 166.67</td>
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</tbody>
</table>

Note: Inc-Increase of energy absorption indices compared with the TH.
**Table 2** Comparison of MCF values between the theoretical and numerical/experimental results.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Theoretical prediction (kN)</th>
<th>Num. or Exp. (kN)</th>
<th>Difference (%)</th>
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<tr>
<td>TH</td>
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<td>3.22 (Num)</td>
<td>-1.86</td>
</tr>
<tr>
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<td>6.85 (Num)</td>
<td>+11.24</td>
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<td>9.11 (Num)</td>
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<td>10.66 (Exp)</td>
<td>+7.41</td>
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<tr>
<td>W/T</td>
<td>d(mm)</td>
<td>Response parameters</td>
<td>Collapse mode</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
<td>---------------------</td>
<td>---------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MCF (kN)</td>
<td>SEA (kJ/kg)</td>
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<tr>
<td>Group A</td>
<td>30</td>
<td>110</td>
<td>11.51</td>
</tr>
<tr>
<td></td>
<td>22.5</td>
<td>110</td>
<td>20.56</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>110</td>
<td>32.31</td>
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<tr>
<td></td>
<td>15</td>
<td>110</td>
<td>44.99</td>
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<tr>
<td></td>
<td>12.86</td>
<td>110</td>
<td>57.27</td>
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<td></td>
<td>11.25</td>
<td>110</td>
<td>70</td>
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<td>Group B</td>
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<td>11.46</td>
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<td>132</td>
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<td>Group C</td>
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<tr>
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<td>154</td>
<td>20.40</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>154</td>
<td>32.53</td>
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<td>154</td>
<td>44.78</td>
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<td>154</td>
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<td>Group D</td>
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<td>31.44</td>
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<td>175</td>
<td>45.11</td>
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<td>11.25</td>
<td>175</td>
<td>31.76</td>
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Table 4 Crashworthiness performances of CTS3 with various W/T under different boundary conditions.

<table>
<thead>
<tr>
<th>W/T</th>
<th>Mass (g)</th>
<th>PCF (kN)</th>
<th>MCF (kN)</th>
<th>SEA (kJ/kg)</th>
<th>EA (J)</th>
<th>CFE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w/o</td>
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<td>w/o</td>
<td>w</td>
<td>w/o</td>
<td>w</td>
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<tr>
<td>8.18</td>
<td>481.14</td>
<td>159.99</td>
<td>161.50</td>
<td>56</td>
<td>110.38</td>
<td>12.73</td>
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<tr>
<td>10</td>
<td>393.66</td>
<td>125.01</td>
<td>125.01</td>
<td>83.95</td>
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<td>12.86</td>
<td>306.18</td>
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<td>87.64</td>
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<td>18</td>
<td>218.70</td>
<td>50.21</td>
<td>50.21</td>
<td>32.31</td>
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<td>22.5</td>
<td>174.96</td>
<td>33.98</td>
<td>33.98</td>
<td>20.56</td>
<td>20.92</td>
<td>12.93</td>
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<td>30</td>
<td>131.22</td>
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<td>20.64</td>
<td>11.51</td>
<td>11.77</td>
<td>9.65</td>
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</tbody>
</table>

Note: w-with bottom clamped; w/o- without bottom clamped.
Table 5 Crashworthiness indices of CTS3 under different crushing velocities.

<table>
<thead>
<tr>
<th>Velocity (m/s)</th>
<th>PCF (kN)</th>
<th>MCF (kN)</th>
<th>SEA (kJ/kg)</th>
<th>EA (J)</th>
<th>CFE</th>
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<tbody>
<tr>
<td>10</td>
<td>24.87</td>
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<td>15.46</td>
<td>12.96</td>
<td>1701.01</td>
<td>0.46</td>
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<td>40</td>
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<td>14.14</td>
<td>1855.03</td>
<td>0.46</td>
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</table>
Table 6 Comparison of the performances of CTS3 and the typical concave structures.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Deformation mode</th>
<th>Mass (g)</th>
<th>EA (J)</th>
<th>SEA (kJ/kg)</th>
<th>MCF (kN)</th>
<th>CFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CST3</td>
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<td>131.22</td>
<td>1266.35</td>
<td>9.65</td>
<td>11.51</td>
<td>0.56</td>
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<tr>
<td>TCT1 Ref.[60]</td>
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<td>131.22</td>
<td>1004.68</td>
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<td>579.338</td>
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<td>0.29</td>
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Table 7 The validation of optimization solutions and numerical results for the CTS3.

<table>
<thead>
<tr>
<th>Case</th>
<th>W (mm)</th>
<th>T (mm)</th>
<th>PCF (kN)</th>
<th>SEA (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Opt result</td>
<td>Num result</td>
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<td>Point B</td>
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<td>Knee point</td>
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<td>0.88</td>
<td>39.31</td>
<td>38.93</td>
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</table>