Fixed-time leader-following flocking and collision avoidance of multi-agent systems with unknown dynamics

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Abstract: The fixed-time flocking of multi-agent systems with a virtual leader is investigated in this paper. The motion dynamics of the agents are assumed to be unknown, which does not need to be modelled by the Lipschitz condition, but only satisfy the boundedness. To achieve the flocking and collision avoidance for all agents in the fixed time, a control protocol in the high-dimensional space is developed by using the graph theory and the theoretical properties of differential equations. Moreover, the upper bound of the settling time only depending on the control protocol and the topology of network is estimated. Numerical examples are used to verify the theoretical results, and show that the proposed method provides an applicable method for the control of the nonlinear dynamical systems.

Keywords: multi-agent systems; differential equations; fixed-time flocking; unknown dynamics; collision avoidance.

1. Introduction

In recent years, the flocking control of multi-agent systems has attracted a considerable amount of attention. A lot of efforts have been paid to understand how a school of fish, a crowd of people, a group of birds, and swarming of bacteria can cluster in formations without centralized coordination (Liu and Jiang, 2020; Mu and He, 2019; Zheng et al., 2020). In the real world problems, learning the mechanism of flocking in biological groups may help to develop many artificial autonomous systems such as crowd evacuation, transport management, and formation control of the intelligent vehicles, the unmanned air vehicles or the autonomous surface vehicles (Dong et al., 2014; Hu et al., 2018). Furthermore, the issue of distributed estimation and control for mobile sensor networks can be addressed based on the flocking algorithm (Su et al., 2016; Su et al., 2017).

Reynolds (1987) first defined the flocking model with three basic rules: Separation, Alignment and Cohesion. Based on this model, a lot of research results have been published in the past decades. Olfati (2006) constructed a smooth potential function acted as the repulsive force or attractive force to satisfy the flocking model, which can guarantee the continuity of energy when the topology is switching. Su et al. (2009a) proposed a leader-following flocking control via pinning strategy. The control protocols nonlinear velocity measurements were considered for the flocking of the agents (Wen et al., 2012; Yang et al., 2010). When the velocity information is unavailable, Su et al. (2009b) investigated the flocking control only based on position measurements. The works (Sun et al., 2019; Zhang et al., 2019) employed the adaptive control protocols to overcome the effect of nonlinear dynamics in multi-agent systems. For the nonlinear dynamics related to all state variables, Su et al. (2013) investigated the adaptive flocking problems governed by locally Lipschitz nonlinearity. Moreover, a novel pinning control for the fragmentation of the network was addressed in Gao et al. (2017). For the directed switching topologies, Wen and Zheng (2018) used the nonsmooth protocols to achieve the coordination of multi-agent systems. The multiple

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Lyapunov functions are constructed in the stability analysis, and it is proven that the convergence can be ensured with the suitable average dwell time.

It is noted that the most existing results for the flocking used different potential functions to steer the agents, and can achieve the flocking asymptotically when time approaches to infinity. However, it is impossible to get the infinity time in the real problems. Therefore, it has more practical significance and more challenging to consider the finite-time convergence for the flocking problems.

There are some similar studies for the fixed-time control problems of multi-agent systems recently, e.g., the fixed-time consensus (Deng et al., 2019) and the fixed-time synchronization (Tan et al., 2020). Ning et al. (2017a, 2017b) focused on the finite-time leader-following consensus with discontinuous dynamics. Gao et al. (2019) investigated the fixed-time stabilization for nonlinear multi-agent systems with the dead-zone input nonlinearity. Moreover, the results (Wang et al., 2020; Zhang et al., 2018) considered the fixed-time synchronization with time-delay. Recently, the general finite-time stability analysis approaches were considered for Takagi–Sugeno fuzzy multi-agent systems (Zhang et al., 2018) and the optimization of energy-cost performance (Zhang et al., 2020) via sliding-mode control.

In the aforementioned works, the analysis of fixed-time problems often utilized the properties of differential equations. Since the potential functions are employed to address the position relationships of the agents in the flocking problems, it is hard to construct such Lyapunov analysis for the Reynolds flocking model. An emergent model without potential function was proposed in Cucker and Smale (2007). Then, such analysis has promoted the development of flocking in a finite time. The flocking problems (Nie and Liu, 2020; Zhang et al., 2020) were investigated based on the C-S type model. In Liu et al. (2020) and Xiao et al. (2020), the fixed-time bipartite flocking was proposed for the multi-agent systems. The fixed-time flocking of Reynolds model with discontinuous dynamics was constructed in Xu et al. (2020). However, compared with the fixed-time results of the multi-agent systems, few works consider the flocking problems in the fixed time or finite time.

Motivated by the above analysis, this paper focuses on the fixed-time flocking model of the multi-agent systems with unknown nonlinear dynamics. The main contributions in comparison with the existing results are as follows: (1) The unknown nonlinear dynamics for each agent only need to be bounded, and do not need to be modelled by the Lipschitz-like condition. (2) A control protocol in the high-dimensional space is developed to achieve the flocking while avoiding the collision, where the design of the control can weaken the effects of unknown dynamics and take effect for different practical applications. (3) A high-dimensional Lyapunov analysis is constructed to prove that the stable flocking within a fixed time and collision avoidance.

The outline of this paper is organized as follows: Section 2 gives the preliminaries and we formulate the problems in Section 3. The main results are provided in Section 4. Numerical experiments are shown in Section 5 to verify the proposed theorem. Finally, Section 6 concludes this work.

2. Preliminaries

In the multi-agent systems, consider n agents moving in a m-dimensional Euclidean space. Let $G(t)=\{v,e(t)\}$ be an undirected graph, where the set of nodes $v=\{1,2,\cdots,n\}$ is the labels of the agents, and the set of edges $e(t)=\{(i,j)\in v\times v\}$ represents the neighboring relations among the agents at time t. $A(G)=(a_{ij})_{n\times n}$ is the adjacent matrix of graph $G(t)$ with elements $a_{ij}=1$ if $(i,j)\in e(t)$ and $a_{ij}=0$ otherwise. Define the Laplacian matrix as $L(G)=D(G)-A(G)$, where $D(G)$ represents the degree matrix of the graph and its $i-th$ diagonal element is the degree of the node i. It is clear that $L(G)$ is a symmetric and positive semi-definite matrix. An alternating sequence of distinct vertices and edges generates a path in the graph. The undirected graph is called connected if and only if there is a path between any pair of distinct nodes.

Lemma 1 (Hong et al., 2006). If L is the symmetric Laplacian matrix of the connected undirected graph G, and the matrix $H = diag(h_1, h_2, \ldots h_n)$ with $h_i \geq 0$ for $i = 1, 2, \ldots, n$, and at least one element in H is positive, then all eigenvalues of the matrix $L + H$ are positive.

Lemma 2 (Valcher and Misra, 2014). If $\xi_1, \xi_2, \ldots, \xi_n \geq 0$, then

$$\sum_{i=1}^{n} \xi_i^{\delta} \geq (\sum_{i=1}^{n} \xi_i)^{\delta} \text{ for } 0 \leq \delta \leq 1 \text{ and } \sum_{i=1}^{n} \xi_i^{\delta} \geq n^{1-\delta} (\sum_{i=1}^{n} \xi_i)^{\delta} \text{ for } \delta > 1.$$

Lemma 3. For a m-dimensional vector $x = [x_1, x_2, \ldots, x_m]^{\mathrm{T}}$, if $sign(x) = sign(x^{\alpha})$, then

$$x^{\mathrm{T}} x^{\alpha} \geq (x^{\mathrm{T}} x)^{\frac{\alpha+1}{2}} \text{ for } 0 \leq \alpha \leq 1 \text{ and } x^{\mathrm{T}} x^{\alpha} \geq m^{1-\frac{\alpha+1}{2}} (x^{\mathrm{T}} x)^{\frac{\alpha+1}{2}} \text{ for } \alpha > 1.$$

Proof. By lemma 2, for $0 \leq \alpha \leq 1$, we have

$$\begin{aligned} x^{\mathrm{T}} x^{\alpha} &= [x_1, x_2, \ldots, x_m][x_1^{\alpha}, x_2^{\alpha}, \ldots, x_m^{\alpha}]^{\mathrm{T}} = x_1^{\alpha+1} + x_2^{\alpha+1} + \cdots + x_m^{\alpha+1} \\ &= (x_1^2)^{\frac{\alpha+1}{2}} + (x_2^2)^{\frac{\alpha+1}{2}} + \cdots + (x_m^2)^{\frac{\alpha+1}{2}} \geq (x_1^2 + x_2^2 + \cdots + x_m^2)^{\frac{\alpha+1}{2}} \\ &= (x^{\mathrm{T}} x)^{\frac{\alpha+1}{2}}, \end{aligned}$$

and for $\alpha > 1$, it holds that

$$\begin{aligned} x^{\mathrm{T}} x^{\alpha} &= [x_1, x_2, \ldots, x_m][x_1^{\alpha}, x_2^{\alpha}, \ldots, x_m^{\alpha}]^{\mathrm{T}} = x_1^{\alpha+1} + x_2^{\alpha+1} + \cdots + x_m^{\alpha+1} \\ &= (x_1^2)^{\frac{\alpha+1}{2}} + (x_2^2)^{\frac{\alpha+1}{2}} + \cdots + (x_m^2)^{\frac{\alpha+1}{2}} \geq m^{1-\frac{\alpha+1}{2}} (x_1^2 + x_2^2 + \cdots + x_m^2)^{\frac{\alpha+1}{2}} \\ &= m^{1-\frac{\alpha+1}{2}} (x^{\mathrm{T}} x)^{\frac{\alpha+1}{2}}. \end{aligned}$$

Lemma 4 (Zuo and Tie, 2014). Given scalar system

$$\dot{z} = -\beta z^{\frac{\varepsilon}{r}} - \gamma z^{\frac{p}{q}},$$

where $\beta, \gamma > 0$ and ε, r, p, q are all positive integers, if $\varepsilon > r$ and $q > p$, then for any initial state $z(0)$, it has $\lim_{t \to T} z(t) = 0$ and $z(t) = 0$ for $t \geq T$, where T is bounded by

$$T \leq \frac{1}{\beta} \frac{r}{\varepsilon - r} + \frac{1}{\gamma} \frac{q}{q - p}.$$

3. Problems formulation

The motion of each agent with a virtual leader is described by

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = u_i + f(p_i) \quad i = 1, 2, \cdots, n \end{cases} \tag{3.1}$$

where $q_i, p_i, u_i \in \mathrm{R}^m$ are the position, velocity and control input vectors of agent i, respectively. $f(p_i)$ is the unknown dynamics in the motion. Assume that each agent has the same sensing radius $r > 0$. The edges between any two agents are generated by $e(t) = \{(i, j) : \| q_i(t) - q_j(t) \| < r, i, j \in v\}$.

Most existing works modeled the unknown dynamics by the Lipschitz condition or Lipschitz-like condition (Sun et al., 2019; Zhang et al., 2019). Such restrictions are very strict and cannot be satisfied in many real problems. In this paper, we only consider the unknown dynamics $f(p_i)$ are only bounded.

Assumption 1: The unknown dynamics $f(\cdot)$ satisfies

$$\| f(x) - f(y) \|_2 \leq \theta, \quad \forall x, y \in \mathrm{R}^m \tag{3.2}$$

where θ is a positive constant.

Remark 1: In a strictly mathematical way, the Lipschitz condition indicates that the function needs to be continuous and has finite derivatives. Compared with the Lipschitz condition, the Assumption 1 on unknown dynamics does not require the continuity and finite derivatives of function, and only needs to be bounded.

The virtual leader is as a global information for each agent and governed by

$$\begin{cases} \dot{q}_\gamma = p_\gamma, \\ \dot{p}_\gamma = f(p_\gamma) \end{cases} \tag{3.3}$$

where $q_\gamma, p_\gamma \in \mathrm{R}^m$ are the position and velocity of the virtual leader, and $f(p_\gamma)$ has the same definition in (3.1).

Definition 1. For the multi-agent systems with a virtual leader, the fixed-time of second-order flocking is achieved if the following conditions are satisfied

(1) $\lim_{t\to T}\max_i \| p_i(t) - p_\gamma(t) \|_2 = 0$ and $\max_i \| p_i(t) - p_\gamma(t) \|_2 = 0,\ \forall t \ge T$.

(2) $\sup_{t\ge 0} \max_{i,j} \| q_i(t) - q_j(t) \|_2 < \infty$.

(3) $\inf_{t\ge 0} \min_{i,j} \| q_i(t) - q_j(t) \|_2 > 0$.

4. Main results

The purpose of this paper is to design a control protocol to achieve the fixed-time flocking with the unknown dynamics. Hence, the algorithm based on the neighbor is employed as

$$\begin{aligned} u_i = & -\kappa sign(\sum_{j\in N_i(t)} a_{ij}(p_i - p_j) + h_i(p_i - p_\gamma)) - (\sum_{j\in N_i(t)} a_{ij}(p_i - p_j) + h_i(p_i - p_\gamma)) \times \\ & [\kappa_1 | \sum_{j\in N_i(t)} a_{ij}(p_i - p_j) + h_i(p_i - p_\gamma) |^{\frac{1}{s}} + \kappa_2 | \sum_{j\in N_i(t)} a_{ij}(p_i - p_j) + h_i(p_i - p_\gamma) |^{-\frac{1}{w}}] \end{aligned} \tag{4.1}$$

where $h_i = 1$ if agent i can receive the information of the virtual leader, and $h_i = 0$ otherwise. κ_1, κ_2, s, w are the positive constants, and κ is a positive parameter to be determined. $N_i(t)$ is the neighbor of agent i at the instant t, which is defined as $N_i(t) = \{ j \mid (i, j) \in e,\ j \ne i,\ j = 1, 2, ..., n \}$.

Define the Lyapunov function of the system as

$$V = \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i(t)} a_{ij} \| p_i - p_j \|_2^2 + \frac{1}{2}\sum_{i=1}^{n} h_i (p_i - p_\gamma)^{\mathrm{T}} (p_i - p_\gamma). \tag{4.2}$$

Theorem 1: Given system (3.1) of n agents with connected undirected topology network G. Each agent is steered by the control protocol (4.1), and the fixed-time flocking can be achieved if the parameter κ in (4.1) satisfies $\kappa \ge \theta\sqrt{n}$. Moreover, the settling time is estimated by

$$T \le T^* = \frac{\kappa_2 s m^{\frac{1}{2s}} n^{\frac{1}{2s}} + \kappa_1 w}{\kappa_1 \kappa_2 \lambda_{\min}(L + H)},$$

and no collision happens during the evolution when the initial position satisfies

$$\min_{i,j} \| q_i(0) - q_j(0) \|_2 > 4T^* V(0)^{\frac{1}{2}} .$$

Proof.

(i). We first prove the convergence of velocities.

Denote the position and velocity differences between the agents and the virtual leader as $\tilde{q}_i = q_i - q_\gamma, \tilde{p}_i = p_i - p_\gamma$, then the motion of each agent can be written as

$$\dot{\tilde{q}}_i = \tilde{p}_i, \dot{\tilde{p}}_i = u_i + f(p_i) - f(p_\gamma),$$

where the control input u_i becomes

$$u_i = -\kappa sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i) - (\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i) \times \\ [\kappa_1 |\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i|^{\frac{1}{s}} + \kappa_2 |\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i|^{-\frac{1}{w}}] \tag{4.3}$$

According the property of Laplacian matrix, the Lyapunov function V can be written as

$$V = \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i(t)} a_{ij} \| \tilde{p}_i - \tilde{p}_j \|_2^2 + \frac{1}{2}\sum_{i=1}^{n} h_i \tilde{p}_i^{\mathrm{T}} \tilde{p}_i = \frac{1}{2}\tilde{p}^{\mathrm{T}} E \tilde{p}. \tag{4.4}$$

where $\tilde{p} = [\tilde{p}_1^{\mathrm{T}}, \tilde{p}_2^{\mathrm{T}}, ..., \tilde{p}_n^{\mathrm{T}}]^{\mathrm{T}} \equiv col(\tilde{p}_1, \tilde{p}_2, ..., \tilde{p}_n) \in \mathrm{R}^{nm}$ and $E = L + H$. Clearly, V is positive semi-definite.

Due to the symmetry of $A(G)$, the time derivative of V along the trajectories of the agents satisfies

$$\dot{V} = \sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(u_i + f(p_i) - f(p_\gamma))]. \tag{4.5}$$

Bring the control input (4.3) into (4.5) that

$$\begin{aligned}\dot{V} = &-\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(\kappa_1(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i) \times |\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i|^{\frac{1}{s}})] \\ &-\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(\kappa_2(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i) \times |\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i|^{-\frac{1}{w}})] \\ &-\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}\kappa sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)] \\ &+\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(f(p_i) - f(p_\gamma))].\end{aligned} \tag{4.6}$$

Noting that

$$\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i = sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i) \times |\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i|, \tag{4.7}$$

and the time derivative of V becomes

$$\begin{aligned}\dot{V} = &-\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(\kappa_1 sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i) \times |\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i|^{1+\frac{1}{s}})] \\ &-\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(\kappa_2 sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i) \times |\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i|^{1-\frac{1}{w}})] \\ &-\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}\kappa sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)] \\ &+\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(f(p_i) - f(p_\gamma))].\end{aligned}$$

It follows from lemma 3 and (4.7) that

$$\begin{aligned}\dot{V} \le &-\sum_{i=1}^{n}\kappa_1 m^{-\frac{1}{2s}}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i]^{1+\frac{1}{2s}} \\ &-\sum_{i=1}^{n}\kappa_2[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i]^{1-\frac{1}{2w}} \\ &-\sum_{i=1}^{n}\kappa[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}} sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)] \\ &+\sum_{i=1}^{n}[(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}}(f(p_i) - f(p_\gamma))]\end{aligned}$$

From lemma 2, there is

$$\begin{aligned}
\dot{V} &\le -\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} [\sum_{i=1}^{n}((\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}} \sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)]^{1+\frac{1}{2s}} \\
&\quad - \kappa_2 [\sum_{i=1}^{n}((\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}} \sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)]^{1-\frac{1}{2w}} \\
&\quad - \kappa \sum_{i=1}^{n} [(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}} sign(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)] \\
&\quad + \sum_{i=1}^{n} [(\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i)^{\mathrm{T}} (f(p_i) - f(p_\gamma))] \qquad (4.8) \\
&\le -\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1+\frac{1}{2s}} - \kappa_2 (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1-\frac{1}{2w}} - \kappa \| E\tilde{p} \|_1 + \| E\tilde{p} \|_2 \| \tilde{f}(p) \|_2 \\
&\le -\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1+\frac{1}{2s}} - \kappa_2 (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1-\frac{1}{2w}} - \kappa \| E\tilde{p} \|_1 + \theta\sqrt{n} \| E\tilde{p} \|_2 \\
&\le -\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1+\frac{1}{2s}} - \kappa_2 (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1-\frac{1}{2w}} - \kappa \| E\tilde{p} \|_1 + \theta\sqrt{n} \| E\tilde{p} \|_1
\end{aligned}$$

where $\tilde{f}(p) = col(f(p_1) - f(p_\gamma), f(p_2) - f(p_\gamma), ..., f(p_n) - f(p_\gamma)) \in \mathrm{R}^{nm}$.

If the parameter κ is chosen as $\kappa \ge \theta\sqrt{n}$, it follows from lemma 1 and (4.8) that

$$\begin{aligned}
\dot{V} &\le -\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1+\frac{1}{2s}} - \kappa_2 (\tilde{p}^{\mathrm{T}} E^{\mathrm{T}} E\tilde{p})^{1-\frac{1}{2w}} \\
&\le -\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} (\lambda_{\min}(E)\tilde{p}^{\mathrm{T}} E\tilde{p})^{1+\frac{1}{2s}} - \kappa_2 (\lambda_{\min}(E)\tilde{p}^{\mathrm{T}} E\tilde{p})^{1-\frac{1}{2w}} \\
&\le -\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} (2\lambda_{\min}(E)V)^{1+\frac{1}{2s}} - \kappa_2 (2\lambda_{\min}(E)V)^{1-\frac{1}{2w}},
\end{aligned}$$

Let $Q = 2\lambda_{\min}(E)V$, one can obtain

$$\dot{Q} \le -2\kappa_1 m^{-\frac{1}{2s}} n^{-\frac{1}{2s}} \lambda_{\min}(E)(Q)^{1+\frac{1}{2s}} - 2\kappa_2 \lambda_{\min}(E)(Q)^{1-\frac{1}{2w}}. \qquad (4.9)$$

Consequently, it can be concluded from lemma 1, lemma 4 and (4.9) that

$$\lim_{t\to T^*} V(t) = 0 \quad \text{and} \quad V(t) = 0 \ \ \forall t \ge T^*,$$

where T^* is estimated by

$$T^* = \frac{\kappa_2 s m^{\frac{1}{2s}} n^{\frac{1}{2s}} + \kappa_1 w}{\kappa_1 \kappa_2 \lambda_{\min}(L+H)} \quad ,$$

From (4.4), $V(t) = 0$ if and only if

$$p_1 = p_2 = \cdots = p_n = p_\gamma,$$

which indicates that

$$\lim_{t\to T^*} \max_i \| p_i(t) - p_\gamma(t) \|_2 = 0 \quad \text{and} \quad \max_i \| p_i(t) - p_\gamma(t) \|_2 = 0, \ \ \forall t \ge T^*.$$

(ii) Now, we consider the distance between any two agents.

According to the Cauchy-Schwartz inequality, the distance difference between any two agents i and j has such differential inequality

$$\begin{aligned}
\frac{d}{dt} \| q_i - q_j \|_2^2 &= 2 < q_i - q_j, p_i - p_j > \\
&\le 2 \| q_i - q_j \|_2 \| p_i - p_j \|_2 .
\end{aligned} \qquad (4.10)$$

Simultaneously, the following equation also holds

$$\frac{d}{dt}\| q_i - q_j \|_2^2 = 2\| q_i - q_j \|_2 \frac{d}{dt}\| q_i - q_j \|_2 . \quad (4.11)$$

It follows from (4.10) and (4.11) that

$$\frac{d}{dt}\| q_i - q_j \|_2 \le \| p_i - p_j \|_2 . \quad (4.12)$$

Integrating the differential inequality (4.12) from 0 to t that

$$\| q_i(t) - q_j(t) \|_2 - \| q_i(0) - q_j(0) \|_2 \le \int_0^t \| p_i(\tau) - p_j(\tau) \|_2 \, d\tau . \quad (4.13)$$

Combining with (4.9), the proof of part (i) implies that the $V(t)$ is a non-increasing function such that

$$V(t) \le V(0), \forall t > 0 . \quad (4.14)$$

It follows from (4.2) and (4.14) that

$$\frac{1}{4}\| p_i - p_j \|_2^2 \le V(t) \le V(0) ,$$

which means that

$$\| p_i - p_j \|_2 \le 4V(0)^{\frac{1}{2}} .$$

Hence, for $t < T^*$, the inequality (4.13) becomes

$$\| q_i(t) - q_j(t) \|_2 - \| q_i(0) - q_j(0) \|_2 \le \int_0^t \| p_i - p_j \|_2 \, d\tau \le \int_0^{T^*} \| p_i - p_j \|_2 \, d\tau \le 4T^* V(0)^{\frac{1}{2}} < \infty$$

and if $t \ge T^*$, it has

$$\| q_i(t) - q_j(t) \|_2 - \| q_i(0) - q_j(0) \|_2 \le \int_0^t \| p_i - p_j \|_2 \, d\tau = \int_0^{T^*} \| p_i - p_j \|_2 \, d\tau + \int_{T^*}^t 0 d\tau \le 4T^* V(0)^{\frac{1}{2}} < \infty.$$

Therefore

$$\sup_{t \ge 0} \max_{i,j} \| q_i(t) - q_j(t) \|_2 < \infty .$$

Furthermore, for $\forall t > 0$, one can get

$$\begin{aligned} \| q_i(t) - q_j(t) \|_2 &= \| q_i(0) - q_j(0) \|_2 - (\| q_i(0) - q_j(0) \|_2 - \| q_i(t) - q_j(t) \|_2) \\ &> \| q_i(0) - q_j(0) \|_2 - |(\| q_i(0) - q_j(0) \|_2 - \| q_i(t) - q_j(t) \|_2)| \\ &= \| q_i(0) - q_j(0) \|_2 - |(\| q_i(t) - q_j(t) \|_2 - \| q_i(0) - q_j(0) \|_2)| \\ &\ge \min_{i,j} \| q_i(0) - q_j(0) \|_2 - 4T^* V(0)^{\frac{1}{2}}. \end{aligned}$$

When the initial position satisfies

$$\min_{i,j} \| q_i(0) - q_j(0) \|_2 > 4T^* V(0)^{\frac{1}{2}} ,$$

there is

$$\| q_i(t) - q_j(t) \|_2 > 0, \forall i, j ,$$

which indicates that no collision happens during the evolution. The proof is completed. □

Remark 2: It is noted that the design of control protocol (4.1) allows the parameter s, w to be selected as any positive numbers. However, only the odd parameters can work in the proposed algorithm by Xiao et al. (2020) and Xu et al. (2020) take effect. The proposed control is more effective for the practical applications.

Remark 3: Compared with the results (Xiao et al., 2020; Xu et al., 2020), the Lyapunov analysis is based on the high-dimensional space and the proof of collision avoidance is given by the strictly mathematical way.

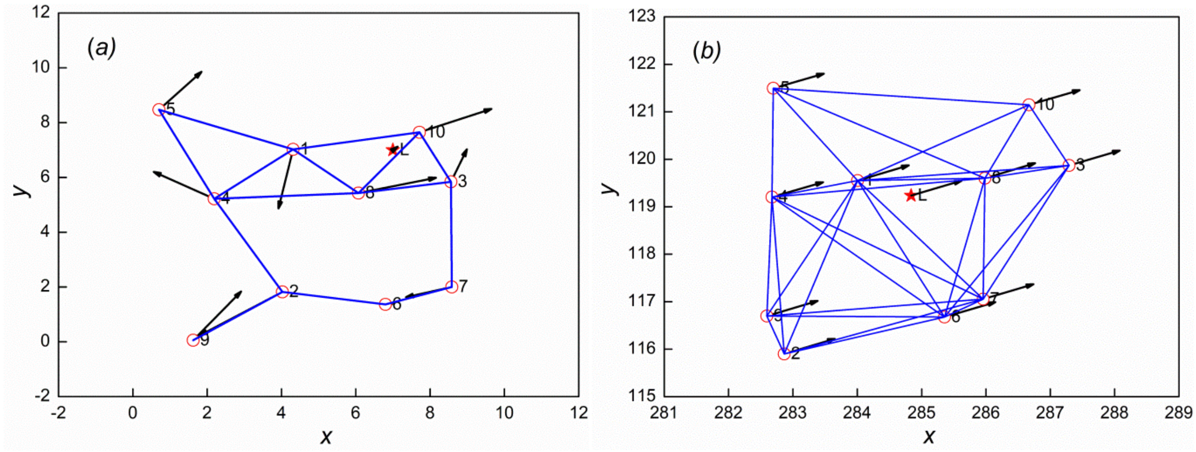

Fig. 1. *The initial and final states of the multi-agent systems.*

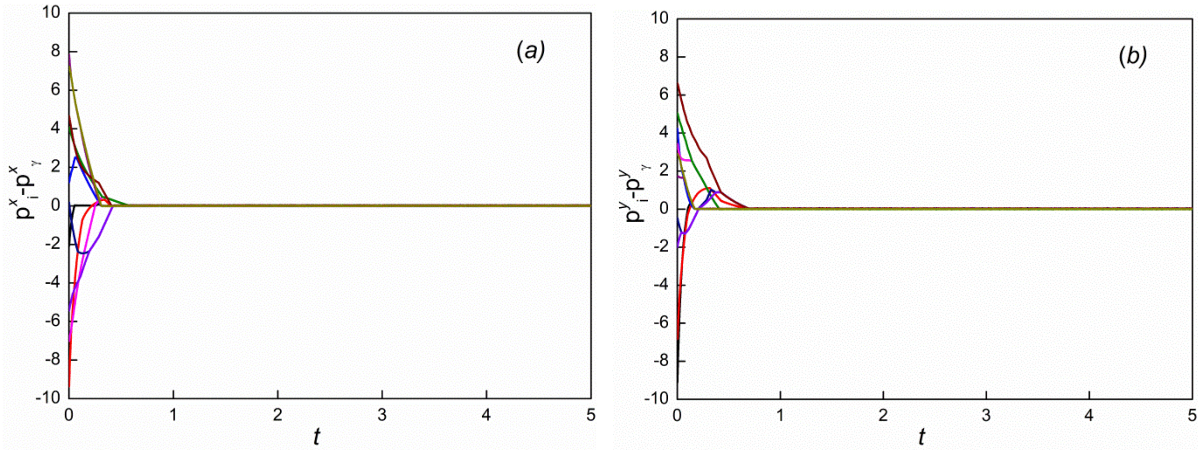

Fig. 2. *Velocity differences between the agents and the virtual leader on x direction and y direction.*

5. Numerical examples

We give a numerical example to illustrate the motion of 10 agents moving in the two-dimensional space. The initial positions of the agents are shown in Fig. 1(a) and the velocities are chosen randomly in the domain of $[-10,10]\times[-10,10]$. The sensing radius is $r=4$ and the neighbor relationships are represented by the solid lines. Each agent is steered by the control input (4.1), where the parameters are $\kappa=7, \kappa_1=1, \kappa_2=3, s=3$ and $w=2$. As the feedback information, the initial position and velocity of the virtual leader are set as $q_\gamma(0)=[7,7]^{\mathrm{T}}$ and $p_\gamma(0)=[0.5,0.5]^{\mathrm{T}}$. Let agent 1 and agent 2 can receive its information. The unknown dynamics $f(p_i)$ is simulated by the following forms

$$f(p_i)=[p_{i2}\sin(\omega_1 p_{i1}), p_{i1}\cos(\omega_2 p_{i2}^2)]^{\mathrm{T}},$$

where ω_1 and ω_2 are the random numbers. It is obvious that such dynamics cannot satisfy the Lipschitz condition.

The final configuration is depicted in Fig. 1(b), which shows that all agents can keep the same velocity with the virtual leader and maintain the finite distance. Fig. 2(a) and Fig. 2(b) show the velocity differences between the agents and the virtual leader on the *x* and *y* axis. It is shown that all the agents will converge to the velocity of the virtual leader in the finite time, and that the design of parameter κ can overcome the influence of unknown dynamics. Moreover, the maximum distance and the minimum distance are descripted in Fig. 3. It is further proved that the evolution of the multi-agent systems can keep the bounded distance and avoid collision. Hence,

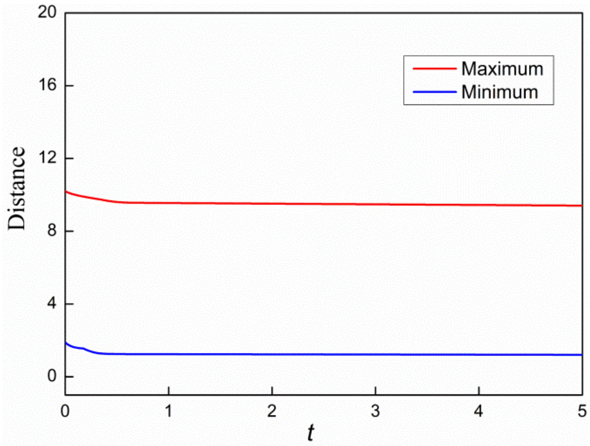

***Fig. 3**. The maximum distance and the minimum distance between any two agents in the motion.*

the main results of this paper can be effectively demonstrated through these numerical simulations.

6. Conclusions

In this paper, the fixed-time flocking of unknown multi-agent systems with a virtual leader is presented. A general control protocol is constructed to achieve the fixed-time flocking in high-dimensional space, where the control protocol is not constrained by the parameters. The proof of the stability of the flocking in the fixed time is given based on the graph theory and Lyapunov method. For a connected undirected network, it can be concluded that the agents can keep the common velocity and the bounded distance in the finite time if a suitable parameter is chosen for the control protocol. Furthermore, the collision during the evolution can be avoided and the upper of the settling time can be estimated. Finally, the simulations illustrate velocity and position curves, which prove the effectiveness of this algorithm.

Our future works will focus on the following two topics. It is noted that the energy function constructed in the analysis needs to utilize the symmetry of adjacency matrix. Once the topology becomes directed, the design of control protocol and energy function for fixed-time flocking may fail to work. For this issue, we will investigate the new control for the fixed-time flocking with directed topology. Moreover, if the nonlinear dynamics are both related to the position and velocity, there needs a new design of the control protocol and energy function to guarantee the similar fixed-time analysis. This problem is complicated and more challenging. In our future work, we will focus on the control protocol to construct the fixed-time flocking while the nonlinear dynamics are both related to the position and velocity.

Conflict of interest

The authors declare that there is no conflict of interest.

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