Uncertainty Analysis and Optimization of Automotive Driveline

Torsional Vibration with Driveline and Rear Axle Coupled Model

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Abstract

The driveline torsional vibration issue is one of the most significant NVH (Noise, Vibration and Harshness) problems, especially in a rear wheel drive vehicle with a manual transmission. In this paper, a new driveline and rear axle coupled torsional vibration model (DRCTVM) is developed, which considers the relationship between the driveline and the rear axle. The experiments have shown that the DRCTVM can provide much better results compared with the traditional model. In addition, for the first time, the uncertainty theory is introduced to the analysis and optimization of driveline torsional vibration based on DRCTVM. A truncated normal distribution is used to describe the uncertainty of DRCTVM, which considers both the probability distribution and bounds of uncertain variables. Furthermore, the robustness of the driveline torsional vibration is analyzed by Monte Carlo process and optimized by Multi-Island Genetic Algorithm. The optimization results have strongly validated that the proposed model and method are very effective to improve the robustness of driveline torsional vibration performance.

Key words: Driveline, Rear axle, Torsional vibration, Truncated probability model, Robust-based design optimization

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1 Introduction

To cope with increasingly market competition and growing user requirement, automotive manufacturers strive for high NVH (Noise, Vibration and Harshness) performance vehicles. The needs for lightweight and fuel-efficient engines are often contradictory to the customer expectations for NVH refinement, which puts forward higher demands to NVH design engineers. NVH performance design is a vehicle level issue which contains many sub-topics, including body NVH, acoustic package, engine NVH, driveline NVH, and abnormal sound control. Among them, the driveline torsional vibration issue is one of the most significant issues, especially in a rear wheel drive vehicle with a manual transmission.

A typical driveline contains engine, clutch, transmission, drive shafts, universal joints, main reducer, haft shafts and wheels, as shown in Fig. 1. The torsional vibration of driveline is a phenomenon that is caused by the engine cyclical excitation, the universal joint secondary couple (Roffey 2012), the driveshaft imbalance and runout. Among them, the engine cyclical excitation is the most important source of driveline torsion vibration (Xia *et al.* 2013). Due to the strong torsional fluctuation inputs and complex interaction, the driveline torsional vibration can induce boom noise, launch shudder, gear rattle and vibration issues in the vehicle.

Fig. 1: Driveline of a RWD (rear wheel drive) vehicle

Torsional vibration problems in automotive driveline have been studied over twenty years by several researchers (Farshidianfar *et al.* 2000; Farshidianfar *et al.* 2002; Farshidianfar *et al.* 2003; Shi *et al.* 2006; Idehara *et al.* 2016). Both sources

(including engine, universal joint, drive shafts unbalance and out run,) and phenomena (including boom noise, launch shudder, gear rattle and vibration) of driveline torsional vibration are examined. Coderc and Callenaere designed an experimental set-up and a computer software to predict dynamic behavior and sought efficient technical solutions of torsional vibrations of a driveline (Couderc *et al.* 1998). Furthermore, Crowther and Zhang presented torsional finite elements for direct, branched and grounded (boundary) connections, and they developed dynamic models of various passenger vehicle powertrains (Crowther *et al.* 2005). The rattle noise of automotive gearboxes was characterized resulting from the impacts between toothed wheels of unselected gear ratios by Kadmiri et al. (Kadmiri *et al.* 2012). Tang et al. developed a novel simplified torsional vibration dynamic model to study the torsional vibration characteristics of a compound planetary hybrid propulsion system (Tang *et al.* 2017). In addition, the drive shafts' influence to the interior vibration and noise was studied by Qatu et al. (Qatu *et al.* 2011).

However, the coupling interaction between the driveline and rear axle is always ignored in the previous work. The rear axle is one of the most important transfer paths of driveline torsional vibration, which can significantly turn the rotational motion of driveline into horizontal motion of body structure. In addition, the rear axle can also change the modal characteristics and torsional vibration responses of driveline. In this paper, a new driveline and rear axle coupled torsional vibration model (DRCTVM) is developed. The new DRCTVM considers the coupling between the driveline and rear axle. The dynamic equation of the DRCTVM is formulated and the modal characteristics are computed and compared with test results.

Another critical issue of torsional vibration problems is the uncertainty. In the real operation of automotive driveline, there inherently exists a vast quantity of uncertainties arising from the driveline system itself, as well as the environmental and operational conditions. The uncertainties of driveline parameters are inescapable in the process of design, manufacture and operation. These uncertainties may cause the system performance to change or fluctuate. Therefore it is necessary to take the uncertainties into account in the design of the driveline system.

Uncertain model and uncertain optimization method can be traced back to 60 years ago (Dantzig 1955; Freund 1956), and since then there is a surge of research in this area. There are several uncertain models to describe the uncertainty, such as probability model (Stefanou 2009), interval model (Moore 1966; Moens *et al.* 2011), evidence model (James 1976), convex model (Ben 1990), and possibility model (Zadeh 1978). The prevalent numerical methods of uncertainty analysis include Monte-Carlo Simulation (Bucher *et al.* 1989; Naess *et al.* 2008; Hurtado *et al.* 2012), Stochastic Perturbation Method (Dash *et al.* 2012; Rong *et al.* 2012), Spectral Stochastic Method (Adhikari 2011). Lots of research have been devoted to design and optimization algorithms under uncertainty and successful applications in vehicle design, including powertrain mount design (Xie *et al.* 2014), engine torque management (Wehrwein *et al.* 2009), and ride comfort control (Jiang *et al.* 2014).

However, to our best knowledge, there is no systematic study using uncertainty-based design and optimization to reduce driveline torsional vibration. In this paper, the uncertainties of the driveline model are described by truncated probability theory, which considers both the probability distribution and bounds. Several parameters of driveline are used as the design variables and the uncertainty-based optimization are conducted to reduce the torsional vibration.

The paper is outlined as follows. First the mathematic model of the new DRCTVM is developed, including the coupling interaction between the driveline and rear axle. Then the free vibration and forced vibration under engine cycle moment are analyzed in Section 3. In Section 4, the uncertainties of the driveline parameters are described by truncated probability model, and the robust-based design optimization of driveline torsional vibration is conducted. The conclusions are made in the last Section.

2 Development of the driveline and rear axle coupled torsional vibration model (DRCTVM)

In this study, the new DRCTVM of a MPV (Multi-Purpose Vehicle) is developed. The vehicle is front-engine and rear wheel drive. An inline-four engine and a five-speed manual transmission are equipped. In the new DRCTVM, the coupling

relationship between the driveline and the rear axle is considered.

In the MPV, there exists a serious problem about driveline torsional vibration, as shown in Fig.2, which plots the sound pressure with the engine speed at middle right passenger's outer ear. It can be seen from Fig. 2 that the vehicle has the noise peaks at 1049rpm, 1602rpm and 2342rpm when traveling at 3WOT (3rd Gear and Wide Open Throttle). The engine 2nd excited has been proved to play the most important role in this peak. The inner sound pressure curves in other locations of the vehicle have the similar characteristic. In the study, such physical phenomenon will be predicted by the proposed DRCTVM.

Fig. 2: Sound pressure at middle right passenger's outer ear

2.1 Development of the driveline physics model

In the traditional driveline simplified model (Vandenplas *et al.* 2003; Wu *et al.* 2006; Zhang *et al.* 2014), both the input shaft of the main reducer and the differential install directly on the ground, as shown in Fig. 3(a). In fact, these parts install on the rear axle housing, which rotates around the wheel axis, as shown in Fig. 3(b). Thus, the coupling motions of the driveline and the rear axle have a great effect on the torsional vibration states of the driveline. To this end, a new simplified model called DRCTVM for the driveline torsional vibration is developed.

(a) Traditional driveline model

(b) DRCTVM

Fig. 3: The coupling relationship of the driveline and the rear axle housing

In DRCTVM, the coupling relationship of the driveline and the rear axle is considered. Both the input shaft of the main reducer and the differential install on the rear axle housing instead of the ground. Lumped parameter method is used to build the model and each component of the driveline is expressed by inertia, stiffness and damping. Fig. 4 shows the new driveline model. The details of parameters employed in the simulation are shown in Table 1, Table 2 and Table 3. T denotes the engine output torque.

Fig. 4: Torsion simulation model of automotive driveline system

In the development of lumped parameter model, the following assumptions and techniques are used:

1. Cranktrain assembly is modeled by a single inertia with input excitations on the crankshaft;
2. For linearity, clutch is assumed to have single stiffness and modelled as main stage stiffness, clutch hysteresis is also taken into account as equivalent viscous damping;
3. Transmission is considered as single inertia and reflected stiffness and damping at its input;
4. The driveshafts and halfshafts are considered as a single inertia which is located in the end of the shaft (Author *et al.* 1992);
5. The torsional stiffness is computed by finite element method with quadratic tetrahedron elements.

Table 1: Inertia parameters in the driveline model

M1	Inertia of cranktrain assembly, clutch cover and clutch plate	M7	Inertia of differential casing
M2	Inertia of clutch hub and transmission input side	M8	Equivalent inertia of halfshafts
M3	Inertia of transmission output dummy	M9	Effective inertia of wheels
M4	Inertia of transmission output dummy	M10	Effective inertia of vehicle
M5	Equivalent inertia of driveshafts	Ma	Inertia of the rear axle housing

M6	Inertia of rear axle input shaft	around the rear wheels' axis
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Table 2: Stiffness and damping parameters in the driveline model		
K1, C1	Main stage clutch damper stiffness and equivalent viscous damping	
K2, C2	Transmission equivalent stiffness and viscous damping	
K3, C3	Double of the effective stiffness and viscous damping of driveshafts	
K4, C4	Double of the effective stiffness and viscous damping of halfshafts	
K5, C5	Stiffness and viscous damping of rear tires	
Ka, Ca	Stiffness and viscous damping of the rear axle housing around the rear wheels' axis	

Table 3: Drive ratio parameters in the driveline model			
R1	Drive ratio of transmission	R2	Drive ratio of main reducer

The inertia and stiffness parameters and the drive ratio of transmission change with the gear stage. For the 3rd gear, all parameters are listed in Table 4. Among them, all inertia dummies are defined by $1\text{E-}5\text{ kg} \cdot \text{m}^2$. As there is no effective engineering method to compute the viscous damping of the driveline parts, the values of viscous damping are determined by experience and references.

Table 4: The parameter values in the 3 rd gear			
Values of inertia	$(10^{-4}\text{ kg} \cdot \text{m}^2)$	Values of stiffness	$(10^2\text{ Nm} / \text{rad})$
M1	1360.3	K1	5.89
M2	40.7	K2	40.4
M3	0.1	K3	363.6
M4	0.1	K4	89.2
M5	221.1	K5	23.0
M6	11.7	Ka	153.3
M7	174.1		
M8	77.9	Values of stiffness	$(\text{Nm} \cdot \text{s} / \text{rad})$
M9	14120.0	C1	0.75
M10	1192250.0	C2	0.01
Ma	1410.3	C3	0.05

Values of drive ratio		C4	0.10
R1	1.341	C5	1.20
R2	4.3	Ca	0.12

2.2 Formulation of system equation

When the drive ratio is r_k , the inertia, stiffness and damping of the driven shaft can be transformed to the driving shaft based on the law of energy conservation, as show in Fig. 5.

(a) Before the transform

(b) After the transform

Fig. 5: The transform of the driven shaft

In Fig. 5, we have

$$m' = m_1 + m_2 / r_k^2 \quad (1)$$

$$k_1' = k_1 \quad k_2' = k_2 / r_k^2 \quad (2)$$

$$c_1' = c_1 \quad c_2' = c_2 / r_k^2 \quad (3)$$

In the DRCTVM, the rear axle housing is coupled with both rear axle input shaft and differential. The local coupled model is shown in Fig. 6. The rotation directions of m_1 and m_2 are different and the coupled system has the relationship $\omega_1 = \omega_2 - \omega_a$, in which ω denotes the rotation speed. So there are only two degrees of freedom in this model.

Fig. 6: Local coupled model of rear axle housing

Let x represent the rotation displacement. The kinetic energy T , the elastic potential energy U and the dissipated energy D can be computed as

$$T = \frac{1}{2} (m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2 + m_a \dot{x}_a^2) \quad (4)$$

$$U = \frac{1}{2} (k_1 x_1^2 + k_2 x_2^2 + k_a x_a^2) \quad (5)$$

$$D = \frac{1}{2} (c_1 \dot{x}_1^2 + c_2 \dot{x}_2^2 + c_a \dot{x}_a^2) \quad (6)$$

The rotation displacement has the relationship of

$$x_a = x_2 - x_1 \quad (7)$$

Apply Eqs. (4) ~ (7) to the Lagrange Equation, we have

$$\begin{bmatrix} m_1 & -m_a \\ -m_a & m_a + m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_a & -k_a \\ -k_a & k_a + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_a & -c_a \\ -c_a & c_a + c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \mathbf{0} \quad (8)$$

The above methods are used in the driveline torsional vibration model as shown in Fig. 4. After the transform, a 9 DOF model is built, which contains 10 inertia parameters, 8 stiffness parameters and 8 damping parameters. The new model is shown in Fig. 7.

Fig. 7: Torsion simulation model after drive ration transform and rear axle housing coupled

The equation of the driveline torsional vibration is derived as follows.

$$\mathbf{M}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T} \quad (9)$$

where

$$\boldsymbol{\theta} = [\theta'_1 \quad \theta'_2 \quad \theta'_3 \quad \theta'_4 \quad \theta'_5 \quad \theta'_6 \quad \theta'_7 \quad \theta'_8 \quad \theta'_9]^T \quad (10)$$

$$\mathbf{M} = \begin{bmatrix} M'_1 & & & & & & & & \\ & M'_2 & & & & & & & \\ & & M'_3 & & & & & & \\ & & & M'_4 & & & & & \\ & & & & M'_5 + M'_a & -M'_a & & & \\ & & & & -M'_a & M'_a + M'_6 & & & \\ & & & & & & M'_7 & & \\ & & & & & & & M'_8 & \\ & & & & & & & & M'_9 \end{bmatrix} \quad (11)$$

$$\mathbf{K} = \begin{bmatrix} K_1' & -K_1' & & & & & & & \\ -K_1' & K_1' + K_2' & -K_2' & & & & & & \\ & -K_2' & K_2' + K_3' & -K_3' & & & & & \\ & & -K_3' & K_3' + K_4' & -K_4' & & & & \\ & & & -K_4' & K_4' + K_a' & -K_a' & & & \\ & & & & -K_a' & K_a' + K_5' & -K_5' & & \\ & & & & & -K_5' & K_5' + K_6' & -K_6' & \\ & & & & & & -K_6' & K_6' + K_7' & -K_7' \\ & & & & & & & -K_7' & K_7' \end{bmatrix} \quad (12)$$

$$\mathbf{C} = \begin{bmatrix} C_1' & -C_1' & & & & & & & \\ -C_1' & C_1' + C_2' & -C_2' & & & & & & \\ & -C_2' & C_2' + C_3' & -C_3' & & & & & \\ & & -C_3' & C_3' + C_4' & -C_4' & & & & \\ & & & -C_4' & C_4' + C_a' & -C_a' & & & \\ & & & & -C_a' & C_a' + C_5' & -C_5' & & \\ & & & & & -C_5' & C_5' + C_6' & -C_6' & \\ & & & & & & -C_6' & C_6' + C_7' & -C_7' \\ & & & & & & & -C_7' & C_7' \end{bmatrix} \quad (13)$$

3 Driveline torsional vibration analysis with DRCTVM

3.1 Development of the engine excitation mathematical model

The engine excitation torque is produced by the gas force in the cylinder, the inertia force and the frictional force. Firstly, the output torque of a crank-rod mechanism with a single cylinder is computed.

In a crank-rod mechanism with a single cylinder, the piston equation of motion can be expressed as (Desbazeille *et al.* 2010):

$$X = R \cos \alpha + L \sqrt{1 - \lambda^2 \sin^2 \alpha} \quad (14)$$

where X is the piston displacement, R and L are the crank radius and connecting length, respectively, and $\lambda = R / L$, α is the crankshaft angle.

The motion equation can be expanded in the forms of infinite series. The approximate solution can be expressed as:

$$X \approx R \cos \alpha + L \left\{ 1 - (\lambda^2 \sin^2 \alpha) / 2 \right\} \quad (15)$$

The reciprocating inertial force can be computed as:

$$F_j = -m_j \ddot{x} \approx m_j R \omega^2 (\cos \alpha + \lambda \cos 2\alpha) \quad (16)$$

where m_j represents the piston mass, and ω is the speed of crankshaft.

The gas pressure in the cylinder can be tested on the engine bench, as shown in Fig. 8. It can be seen from Fig. 8 that the cylinder pressure has one peak in a cycle. Hence the output moment of an inline-four engine should have four peaks in a cycle. The gas force F_p can be computed by the product of cylinder pressure and area of cylinder section. The resultant force on the piston can be expressed by $P = F_j + F_p$.

Fig. 8: Cylinder pressure curves in different crankshaft speed

The force of friction is also considered. Let $\varphi = \arctan \mu$ represent the frictional angle, where μ is the frictional coefficient. The circle radius of friction can be computed as:

$$\rho = r\mu / \sqrt{1 + \mu^2} \quad (17)$$

where r is the journal radius of the revolute pair.

Let r_A , r_B and r_C be the journal radius of the crankshaft, big end of the connecting rod and small end of the connecting rod. The friction circle radius at the three locations ρ_A , ρ_B and ρ_C can be easily computed by Eq. (17).

The sketch map of mechanism force analysis is shown in Fig. 9 when $0 \leq \alpha \leq \pi/2$. It can be seen from Fig. 9 that

$$\gamma = \arcsin[(\rho_B + \rho_C) / L] \quad (18)$$

$$h = R \sin(\alpha + \beta - \gamma) - \rho_A - \rho_B \quad (19)$$

where γ is the angle between the connecting rod force line and the line BC, and h denotes the distance between the force F_{21} and F_{41} .

Fig. 9: Sketch map of mechanism force analysis when $0 \leq \alpha \leq \pi / 2$

Based on the sine law, we can obtain the following equation

$$F_{21} = F_{23} = P \cos \varphi / \cos(\beta - \varphi - \gamma) \quad (20)$$

The engine output torque can be computed by $T = F_{21}h$, so

$$T = \frac{P \cos \varphi}{\cos(\beta - \varphi - \gamma)} [R \sin(\alpha + \beta - \gamma) - \rho_A - \rho_B] \quad (21)$$

The output torque can be computed by the same way when $\pi / 2 \leq \alpha \leq \pi$, $\pi \leq \alpha \leq 2\pi / 3$, $2\pi / 3 \leq \alpha \leq 2\pi$.

From the above, the output torque of single cylinder engine in one working cycle ($0 \leq \alpha \leq 4\pi$) can be expressed as:

$$T = \frac{P \cos \varphi [R \sin(\alpha + \beta - \gamma) - \rho_A - \rho_B]}{\cos(\beta - \varphi - \gamma)} \quad (22)$$

where

$$\gamma = \begin{cases} \arcsin \frac{\rho_A + \rho_C}{L} & \alpha \notin \left\{ \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \cup \left[\frac{5\pi}{2}, \frac{7\pi}{2} \right] \right\} \\ \arcsin \frac{\rho_B - \rho_C}{L} & \alpha \in \left\{ \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \cup \left[\frac{5\pi}{2}, \frac{7\pi}{2} \right] \right\} \end{cases} \quad (23)$$

Through the phase combination of different cranks, the excitation torque of the inline-four engine can be computed by

$$T = \sum_{i=0}^3 \frac{(F_p + F_j) \cos \varphi [R \sin(\alpha + i\pi + \beta - \gamma) - \rho_A - \rho_B]}{\cos(\beta - \varphi - \gamma)} \quad (24)$$

where

$$F_p = \pi D^2 p_g (\alpha + i\pi) \quad (25)$$

$$F_j = m_j R \omega^2 [\cos(\alpha + i\pi) + \lambda \cos 2(\alpha + i\pi)] \quad (26)$$

in which D denotes the piston diameter and p_g represents the cylinder pressure.

The engine excitation torque can be computed by Eq. (24), as shown in Fig. 10. As outlined in Fig. 10, there are two peaks in the excitation torque curve when the crankshaft rotates one circle (360 degree). Hence the 2nd order is the most important part of the engine excitation torque.

Fig. 10: Engine excitation torque with the crankshaft angle

3.2 Free vibration analysis of driveline

From Eq. (9), neglecting the damping and excitation terms, the eigensolutions of the systems can be obtained by using the following equation:

$$(\mathbf{K} - \lambda \mathbf{M})\mathbf{A} = 0 \quad (27)$$

where λ are the eigenvalues of the system, and $\mathbf{A}$ stands for the eigenvectors. The modal frequencies of the driveline are listed in Table 5.

Table 5: Modal frequencies of DRCTVM

Mode No.	Modal frequency (Hz)	Mode No.	Modal frequency (Hz)
1	3.5	5	123.4
2	12.0	6	422.5
3	36.4	7	530.5
4	54.4	8	754.4

In order to validate the accuracy of DRCTVM modal shapes, the modal frequencies and shapes are also obtained by the modal test. The installation sites of the torsional vibration sensors are shown in Fig. 10. The engine itself is treated as the exciting source. The rotation speed of engine is from 1000rpm to 5000rpm. Hence the method of excitation is the swept frequency excitation. And the exciting frequency is from about 30Hz to 165Hz. Six sensors are installed on the driveline. Their locations are the flywheel, the input shaft of the transmission, the driveshaft, the input shaft of the main reducer and the half shafts. Because of the limit of the frequency range of the simulated model and test conditions, only the 3rd to 5th modes are tested.

(a) Sensor at input shaft of the transmission

(b) Sensor at drive shaft

(c) Sensor at input shaft of the main reducer

Fig. 11: The installation site of the torsional vibration sensors

Both the modal frequencies obtained from traditional model and DRCTVM are compared with the test values, as shown in Table 6. The modal shapes of simulation and test are also be compared, as shown in Fig. 12.

Table 6: Modal frequencies benchmarking of the driveline system

Mode No.	DRCTVM	Traditional model	Test
3	36.4 Hz	38.3 Hz	34.5 Hz
4	54.4 Hz	—	54.6 Hz
5	123.4 Hz	130.3 Hz	115.2 Hz

(d) 3rd modal shape

(e) 4th modal shape

(f) 5th modal shape

Fig. 12: Simulated modal shapes of the driveline

In Fig. 12, the numbers 1 to 9 denote the degree of freedom at correlative parts, as shown in Table 7.

Table 7: The corresponding parts of the number in Fig. 12

No.	Parts	No.	Parts
1	Engine	6	Rear axle housing
2	Transmission input shaft	7	Halfshafts
3	Transmission output dummy	8	Wheels
4	Transmission output dummy	9	Vehicle
5	Driveshafts		

It can be seen from Table 6 and Fig. 12 that the rear axle has a great influence on the torsional modal of the driveline. Unfortunately, the 4th modal cannot be obtained by the traditional model. That is because that the DRCTVM has 9 degrees of freedom, while the traditional model has 8 degrees of freedom. The number of degrees of freedom is equal to the number of modals. Hence the DRCTVM has one more modal than the traditional model. For DRCTVM, the accessorial modal is the 4th modal. On the contrary, the DRCTVM is very accurate to describe the torsional vibration performance compared with the traditional model. In addition, it is noticed that the 3rd and the 4th modal frequencies agree with the noise peak at 1049rpm and 1602rpm compared with the interior noise as shown in Fig. 2.

It also can be seen from Table 6 that the modal frequency difference between the DRCTVM and the test is less than 7%. Considering the uncertainty and the test errors, the differences can be accepted in practice. Several methods can be used to improve the correlation, such as accurate measurement of the model parameters and application of the finite element model. However, these methods cost a large amount of time and are not suitable for this work.

3.3 Forced vibration analysis of driveline

The directed frequency response method is used to compute the forced vibration of the driveline. By Laplace Transform, the Eq. (9) can be converted to

$$(-\mathbf{M}\omega^2 + j\omega\mathbf{C} + \mathbf{K})\boldsymbol{\theta}(\omega) = \mathbf{T}(\omega) \quad (28)$$

where ω denotes the circle frequency.

Fast Fourier Transform (FFT) is conducted on the dynamic torque to calculate the mean and alternating torque of multiple harmonics of engine firing frequency. The

results of the torque in frequency domain are shown in Fig. 13. It is obvious that the engine excitation has the characteristics of orders, and the 2nd order is particularly remarkable.

Fig. 13: Colormap spectrum of the engine output torque

The 2nd order torque of the engine is used as the excitation source. The angular acceleration at the flywheel, the input shaft of transmission and the input shaft of the main reducer are calculated, as shown in Fig. 14. It can be known from Fig. 14 that the torsional vibration at the transmission and main reducer is much larger than that at the flywheel at about 1100rpm and 1650rpm. Additionally, the 2nd order responses of the driveline simulation model agree with the interior noise in Fig. 3 at the locations of peaks.

Fig. 14: The 2nd order angular acceleration response

Fig. 15 shows the acceleration response curves computed by the traditional model and the new DRCTVM. As expected, the second peak at 1650rpm cannot be obtained by the traditional model. By contrast, the new DRCTVM with consideration of the coupling between the driveline and rear axle and can predict both peaks at 1100rpm and 1650rpm, which agree very well with the interior noise. The results shown in Fig. 15 have strongly demonstrated the advantages of the proposed DRCTVM compared with the traditional model.

Fig. 15: The angular acceleration response comparison between the traditional model and
DRCTVM

4 Uncertainty analysis and optimization of driveline torsional vibration

4.1 Uncertainty of the driveline parameter with truncated probability model

In the process of manufacture and use, the parameters of the driveline model usually fluctuate around its design values. The normal distribution is often used to describe the uncertainty of the manufacture parameters. In the normal distribution, the values of manufacture parameters may be much larger than the design values. However, in engineering practice, the products which have large deviations will be abandoned. Hence a truncated normal distribution is employed to describe the uncertainty of the driveline parameters. (Jiang *et al.* 2012)

For an ideal normal distribution with a PDF (probability density function) $f_x(x)$, its corresponding PDF $f'_x(x)$ of the truncated distribution can be defined by

$$f'_x(x) = \begin{cases} a \cdot f_x(x) & x^L \leq x \leq x^R \\ 0 & x < x^L \text{ or } x > x^R \end{cases} \quad (29)$$

in which x^L and x^R are the lower and upper bounds of the parameter x , and a is an amplification coefficient. The curves of $f_x(x)$ and $f'_x(x)$ are plotted in Fig. 16. It is seen that the truncated distribution not only describes the probability distribution of the uncertain variables, but also considers the interval bounds of the variables.

Fig. 16: The PDF of normal distribution and the corresponding PDF of truncated normal distribution

As the area of the PDF is equal to 1, the coefficient a can be computed by

$$a = \frac{1}{\int_{-\infty}^{x^R} f_x(x) dx - \int_{-\infty}^{x^L} f_x(x) dx} = \frac{1}{F_x(x^R) - F_x(x^L)} \quad (30)$$

where $F_x(x)$ denotes the CDF (cumulative density function). Thus, the corresponding CDF $F'_x(x)$ of the truncated distribution can be computed as

$$F'_x(x) = \begin{cases} 0 & x < x^L \\ \frac{F_x(x) - F_x(x^L)}{F_x(x^R) - F_x(x^L)} & x^L \leq x \leq x^R \\ 1 & x > x^R \end{cases} \quad (31)$$

There are four parameters to determine a truncated normal distribution: expected value μ , standard deviation σ , upper bound x^R and lower bound x^L . For engineering problems, the expected value can be assigned by design value. While the machining tolerance can be used to determine the bounds. However, the bounds are

very hard to obtain in real situations. Hence an equal CDF method is presented here to discuss the value of the bounds (Xu *et al.* 2006). In this method, the cut CDF parts at the two bounds should be the same. Thus, we have the following equation:

$$r = F_X(x^L) = 1 - F_X(x^R) \quad (32)$$

where r is defined as truncating ratio.

Let ϕ^{-1} be the inverse CDF of the standard normal distribution, the upper and lower bounds can be computed based on Eq. (32).

$$\begin{aligned} x^R &= \phi^{-1}(r) \cdot \sigma + \mu \\ x^L &= \phi^{-1}(1-r) \cdot \sigma + \mu \end{aligned} \quad (33)$$

Substituting Eqs. (32) and (31) to Eqs. (30) and (32), the PDF and CDF of the truncated normal distribution can be expressed as

$$f'_X(x) = \begin{cases} \frac{f_X(x)}{1-2r} & x^L \leq x \leq x^R \\ 0 & x < x^L \text{ or } x > x^R \end{cases} \quad (34)$$

$$F'_X(x) = \begin{cases} 0 & x < x^L \\ \frac{F_X(x) - r}{1-2r} & x^L \leq x \leq x^R \\ 1 & x > x^R \end{cases} \quad (35)$$

In the proposed DRCTVM, there are totally 19 parameters, as shown in Table 4. In order to simplify the analysis, only the parameters which have a great influence on the critical modals and responses are considered as the uncertain variables. In this work, the uncertainties of the drive ration and engine excitation are ignored.

The sensitivity of parameter x to the function $f(x)$ is defined as

$$s_x = \beta \left| \frac{f(x + \Delta x) - f(x)}{\Delta x} \right| \quad (36)$$

where Δx is a tiny increment and β is a scale factor. For different kinds of load steps and parameters, the value of β is different. Hence the sensitivity to the critical modals and response curve peak of each parameter is computed and listed in Table 8

and Table 9.

Table 8: The inertia parameter sensitivity

	3 rd mode	4 th mode	1 st peak at gearbox	2 nd peak at gearbox	1 st peak at rear axle	2 nd peak at rear axle
M1	0.54	0.01	35.73	7.78	32.33	19.98
M2	64.57	4.41	10.59	95.11	121.81	244.24
M5	42.02	1.89	135.90	87.38	126.14	148.33
M6	37.57	1.13	161.89	69.40	92.14	99.70
M7	1.61	1.55	5.19	20.14	0.46	52.94
M8	0.60	0.71	1.94	9.33	0.17	24.52
M9	0.07	0.01	0.19	0.13	0.00	0.35
M10	0.00	0.00	0.00	0.00	0.00	0.00
Ma	0.52	18.26	1.71	98.16	4.37	349.31

Table 9: The stiffness parameter sensitivity

	3 rd mode	4 th mode	1 st peak at gearbox	2 nd peak at gearbox	1 st peak at rear axle	2 nd peak at rear axle
K1	15.00	0.43	111.05	24.66	100.47	63.33
K2	0.04	0.00	0.15	0.28	0.55	0.42
K3	0.03	0.01	0.09	0.14	0.15	0.33
K4	0.40	0.15	1.23	1.92	0.08	5.04
K5	0.01	0.00	0.04	0.01	0.00	0.03
Ka	0.10	1.57	0.32	10.28	0.81	36.68

It is obvious that the parameters M2, M5, M6, M7, Ma, K1, and Ka have a significant effect on the critical modals or response peaks. Therefore, the seven parameters are chosen as the uncertain variables. Each uncertain variable is described as a truncated normal distribution. The expected value is assigned as the design value. The standard deviation is estimated as 5% of the expected value. The upper and lower bounds are computed by Eq. (33). All the distribution parameters of the uncertain variables are listed in Table 10.

Table 10: The distribution parameters of the uncertain variables

	expected value	standard deviation	lower bound	upper bound
M2	4.07E-03	2.04E-04	3.66E-03	4.48E-03
M5	2.21E-02	1.11E-03	1.99E-02	2.43E-02
M6	1.17E-03	5.85E-05	1.05E-03	1.29E-03
M7	1.74E-02	8.71E-04	1.57E-02	1.92E-02
Ma	1.41E-01	7.05E-03	1.27E-01	1.55E-01
K1	5.89E+02	2.93E+01	5.30E+02	6.48E+02
Ka	1.53E+04	7.67E+02	1.38E+04	1.69E+04

4.2 Monte Carlo process of driveline torsional vibration

Although the computational cost is very expensive in Monte-Carlo (MC) simulation compared with other uncertainty methods, MC is able to provide much better accuracy. As the computational cost is less than 5s in driveline torsional analysis, MC is applied in this study.

2400 sample points are generated by Monte-Carlo Method. The 3rd and 4th modal frequencies and peaks of the vibration acceleration at gearbox and rear axle are defined as the output variables, as shown in Table 11 and Fig. 17.

Table 11: The distribution parameters of the output parameters

	mean	standard deviation	lower bound	upper bound
3 rd modal frequency	36.38	0.61	34.45	38.58
4 th modal frequency	54.41	1.53	50.13	59.38
1 st peak at gearbox	5104.20	193.73	4600.72	5604.95
2 nd peak at gearbox	1403.53	624.54	577.40	3485.75
1 st peak at rear axle	4585.71	203.82	4032.24	5184.73
2 nd peak at rear axle	4271.80	2154.26	1449.58	9680.15

(a) Distribution of 3rd modal frequency

(b) Distribution of 4th modal frequency

(c) Distribution of 1st peak at gearbox

(d) Distribution of 2nd peak at gearbox

(e) Distribution of 1st peak at rear axle

(f) Distribution of 2nd peak at rear axle

Fig. 17: Distribution of the output variables

It can be seen from Table 11 and Fig. 17 that the peaks at rear axle are very sensitive to the values of driveline parameters. With slight change of driveline parameters, the peaks change dramatically. Hence it is necessary to optimize the driveline torsional vibration performance based on robust-based design.

4.3 Robust-based design optimization and results

Robust-based design describes the degree of tolerance of the system to be insensitive to variations in both the system itself and the environment (Yao *et al.* 2011). Robust-based design optimization is a systematic and efficient way to meet the challenge of design optimization for performance and quality (Ghosh *et al.* 2017), which aims to find the insensitive design space of the design variables and decrease the fluctuation of the target responses under uncertainty. The mathematical

formulation of the robust-based design optimization can be expressed as (Fogliatto 1997):

$$\begin{cases} \text{find} & \mathbf{x} \\ \min & \bar{f}(\mathbf{x}, \mathbf{p}) = F(\mu_f(\mathbf{x}, \mathbf{p}), \sigma_f(\mathbf{x}, \mathbf{p})) \\ \text{s.t.} & \mathbf{g}(\mathbf{x}, \mathbf{p}) \leq 0 \\ & \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^R \end{cases} \quad (37)$$

where $\mathbf{x}$ is the design variable vector, $\mathbf{p}$ is the constant parameter vector, and both $\mathbf{x}$ and $\mathbf{p}$ could be uncertain, f denotes the objective function and $\mathbf{g}$ denotes the constraint vector. $\mathbf{x}^L$ and $\mathbf{x}^R$ are the lower and upper bounds of $\mathbf{x}$.

The objective function can be formed as

$$F(\mu_f(\mathbf{x}, \mathbf{p}), \sigma_f(\mathbf{x}, \mathbf{p})) = \frac{k\mu_f(\mathbf{x}, \mathbf{p})}{w_\mu} + \frac{(1-k)\sigma_f(\mathbf{x}, \mathbf{p})}{w_\sigma} \quad (38)$$

where k is the weighting factor, w_μ and w_σ are the scaling factors.

The parameters of driveline system are limited by many performances, such as dynamic property, fuel economy and general arrangement. Some parameters have a great effect on these performances. Hence not all parameters can be used as optimization variables in the process of torsional vibration optimization design, especially in the late stage of the vehicle development. In this paper, we chose the inertia of the rear axle housing, the torsional stiffness of the main stage clutch damper and the rear axle housing around the rear wheels' axis as the design variables. **And in this work, both $\mathbf{x}$ and $\mathbf{p}$ are uncertain.**

$$\begin{aligned} \mathbf{x} &= [\mathbf{Ma} \quad \mathbf{K1} \quad \mathbf{Ka}]^T \\ \mathbf{p} &= [\mathbf{M2} \quad \mathbf{M5} \quad \mathbf{M6} \quad \mathbf{M7}]^T \end{aligned} \quad (39)$$

Consider the peaks at rear axle as the objective function, and the modal frequencies and the peaks at gearbox are used as the constraint conditions. The bounds and distribution type of the design variables are listed in Table 12.

Table 12: The bounds and distribution type of the design variables

	Distribution type	lower bound	upper bound
Ma	Truncated normal distribution	1.20E-01	1.60E-01
K1	Truncated normal distribution	4.00E+02	9.00E+02
Ka	Truncated normal distribution	1.00E+04	2.00E+04

Multi-Island Genetic Algorithm (Hu *et al.* 2014) is used to optimize the values of the three design variables. The optimal results of design variables and responses can be seen in Table 12. The statistical properties of the optimal results are presented in Table 13 and Table 14. The initial means, standard deviations in Table 13 and the initial bounds in Table 14 are all computed by the Monte Carlo process which is detailed in Section 4.2.

Table 12: The optimal results of design variables and responses

	Initial value	Optimal value
Ma	1.41E-1	1.21E-1
K1	5.89E+2	4.05E+2
Ka	1.53E+4	1.94E+4
3 rd modal frequency	36.4	33.7
4 th modal frequency	54.4	65.2
1 st peak at gearbox	5120.7	3924.4
2 nd peak at gearbox	1018.6	484.8
1 st peak at rear axle	4632.9	3484.2
2 nd peak at rear axle	2615.7	602.6

Table 13: The statistical properties of the optimal results

	Initial mean	Initial standard deviation	Optimal mean	Optimal standard deviation
3 rd modal frequency	36.38	0.61	33.72	0.53
4 th modal frequency	54.41	1.53	65.23	1.93
1 st peak at gearbox	5104.20	193.73	3878.30	139.98
2 nd peak at gearbox	1403.53	624.54	486.19	40.62
1 st peak at rear axle	4585.71	203.82	3456.10	137.30
2 nd peak at rear axle	4271.80	2154.26	606.66	84.60

Table 14: The lower and upper bounds of the optimal results

	Initial lower bound	Initial upper bound	Optimal lower bound	Optimal upper bound
3 rd modal frequency	34.45	38.58	32.08	35.52
4 th modal frequency	50.13	59.38	59.86	70.73
1 st peak at gearbox	4600.72	5604.95	3246.10	4243.50
2 nd peak at gearbox	577.40	3485.75	373.63	814.54
1 st peak at rear axle	4032.24	5184.73	2917.50	3834.60
2 nd peak at rear axle	1449.58	9680.15	478.79	2939.70

From Table 12, it is obvious that the peaks' values after optimization are much smaller than the initial design values. In addition, the means and standard deviations are also much smaller compared with the initial design without optimization from Table 13. Furthermore, it is observed that the variations of modal frequencies and peaks' values become narrow as shown in Table 14 based on robust-based design. These numerical results have clearly demonstrated the effectiveness of the proposed model, which is very important in the real application of design of the driveline torsional system.

5 Conclusions

In this paper, a new driveline and rear axle coupled torsional vibration model (DRCTVM) is developed by installing the input shaft of main reducer and the differential on the rear axle instead of the ground. Therefore, the coupling relationship of the driveline and the rear axle is considered in the proposed DRCTVM. The free vibration and forced vibration characteristics are analyzed and compared with the test results. The test results have indicated that the new DRCTVM has provided much better accuracy compared with the traditional model.

The uncertain analysis and optimization method are conducted to analyze the vibration issue of driveline torsional system. The torsional angular accelerations of transmission input shaft and main reducer input shaft are computed to evaluate the torsional vibration performance. Monte Carlo method and Multi-Island Genetic Algorithm are used to analyze and optimize the uncertainty of the driveline torsional vibration. After optimization, the robustness of the driveline torsional vibration is much better than the initial state. The model and method developed in this work are very effective to improve the robustness of the driveline torsional vibration.

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