Improve impact responses of a honeycomb sandwich panel structure with internal resonators

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Abstract

In this paper, a decahedral honeycomb sandwich panel structure with periodic internal resonators is developed in order to effectively mitigate the impact force and dissipate the impact energy. Five types of local resonance microstructures (LMs) with the property of negative effective mass are applied on the sandwich structure to study the effects of different resonators on load attenuation, and the theoretical and numerical results show that the multi-resonator model (MM) exhibits the best attenuation effect. In addition, the damping characteristic is added to the MM to further improve the structure responses. The analysis results have clearly demonstrated that the combination of damping characteristic and local resonance of multi-resonators can significantly dissipate the impact energy and mitigate the dynamic load. In addition, the optimization analyses are also carried out to optimize the spring stiffness, lumped mass and damping coefficient. The performance of the sandwich panel structure is significantly improved with a relatively small structure mass.

Key words: resonator; local resonance microstructure; honeycomb sandwich structure; impact force; impact energy;

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1. Introduction

Over the past decades, the honeycomb sandwich panel structures are widely used as the reinforcing structure to enhance the energy absorbing in the crash. Many have been conducted to improve the performance of the sandwich panel structures (Hou et al. 2015; Wu, Liu, et al. 2017; Xu, Jiang, and Lee 2017; Yang et al. 2017). example, an explosion test (Dharmasena et al. 2008) was conducted to study the dynamic response of a three-dimensional sandwich panel structure with square honeycomb core composed of austenitic steel alloys. The tests of three-point bending and in-panel compression on one kind of hexahedral aluminum honeycomb sandwich panel (Sun et al. 2017) were performed, and results showed that the collapse mode and structure crashworthiness of the sandwich structure were significantly affected by the structural and adhesive parameters. The mechanical performance of a composite honeycomb sandwich panel was studied through the finite element analysis and three point bending test (Lu et al. 2015), and the failure mode of this panel structure was shown in detail. The mechanical performance of a corrugated core sandwich panel under in-plane compressive load was studied by (Biagi and Bart-Smith 2012), and the failure mechanisms of macro buckling, shear buckling and face wrinkling were identified one by one.

The above studies are mainly focused on the mechanical properties of the sandwich structure and the improvement of the stiffness and the energy absorption efficiency of the sandwich structure. However, the mitigation of impact force is rarely studied. Impact force is a key factor in structural crashworthiness. If the attenuation of the impact force is achieved, the structure can be effectively protected. Moreover, the attenuation of the impact force can also be used to protect the person in the collision, such as the protection of passengers during the crash of vehicles.

In this work, the collision analysis of a decahedral honeycomb sandwich panel structure with internal resonators is carried out to attenuate the impact force. The combination of internal resonators and matrix structures is called the LMs, which can exhibit extraordinary physical properties. These properties include the negative

effective mass (Li, He, Li, et al. 2018; Li, He, Wang, et al. 2018; Xiao et al. 2019), the negative effective bulk modulus (Liu et al. 2011; Wu, Wang, et al. 2017) and the negative effective shear modulus (Lai et al. 2011; Ying, Lai, and Zhang 2011). Among these properties, the negative effective mass obtained by the LMs can be applied to produce the forbidden bands that prevent the propagation of elastic waves. This bizarre phenomenon is due to the coupling effects of the propagating of waves in the host medium and the resonance motion of oscillators.

Previous scholars have conducted in-depth studies on the factors that affect the performance of LMs. The single-resonator microstructure (Jiao et al. 2015) was applied to realize the negative mass density and mitigate the mechanical wave at the local resonance frequency. Following this, the dual-resonator microstructure (Huang and Sun 2009; Tan, Huang, and Sun 2012) was studied to produce the double band gaps, and the multi-resonator microstructure (Li et al. 2019) with more degrees of freedom can generate multiple band gaps to manipulate the mechanical waves better. Consequently, the influences of damping characteristic on the performances of LMs were studied. The wave propagation characteristic in the mass-in-mass damped single-resonator lattice system was studied by (Islam and Newaz 2012; Chen et al. 2016). It was found that the damping can extend the attenuation frequency range of mechanical wave and exhibit the metadamping phenomenon. When the damped oscillators and the beam structure were combined (Thompson 2008), the beam structure can exhibit the mitigation effect of mechanical wave. With high damping coefficients and multiple tuning resonance frequencies, this system can obtain wide regions of attenuation frequency. The global optimization techniques (Bouhlef et al. 2018; Chen et al. 2019; Kadlec and Šeděnka 2018) are also very effective to improve the performance of LMs. The propagation of elastic wave in a beam lattice material characterized by the anti-chiral cell was investigated by (Bacigalupo et al. 2016). The number, location and properties of the oscillators can be designed to expand the width of the band gaps. An in-depth study of LMs composed of rubber-coated inclusions was presented by (Krushynska, Kouznetsova, and Geers 2014). Based on the results of numerical analyses, the optimal microstructure produces the widest frequency band

gap. The underwater low-frequency sound absorption of plate structure with several layers (Meng et al. 2012) was enhanced through using the genetic algorithm and a nonlinear constrained algorithm. The physical parameters of the plate structure were optimized, and the theoretical and experimental results proved that the optimized plate can realize the broadband absorption of underwater sound.

Besides the above separate studies of resonators and sandwich structures, scholars had attempted to combine resonators and sandwich beam structures together to manipulate the wave propagation. For example, the dissipated multi-resonators were applied in the sandwich beam structure (Chen et al. 2017) to further mitigate and absorb the broadband wave. A new type of sandwich beam with the internal oscillators was shown by (Sharma and Sun 2016), and the system can produce the local resonance and Bragg band gaps simultaneously. A new type of broadband vibration absorbing beam based on multi-frequency vibrator was proposed by (Pai, Hao, and Jiang 2014). Theoretical and numerical analyses showed that the dissipated oscillators produce two band gaps that can be applied to absorb and stop the propagation of waves.

In addition to the above theoretical studies, the engineering applications of resonators with the property of negative effective mass on energy absorption were also studied. The polymer matrix with resonant particles (Wang et al. 2017) was designed to absorb the blast wave energy and attenuate the blast wave. Two composite materials with different geometry parameters were proposed to investigate the attenuation effect, and the energy transfer effect between components was studied by calculating the energy history of the system. The performances of metaconcrete under blast and elastic waves were investigated by (Mitchell, Pandolfi, and Ortiz 2014, 2015). Numerical results showed that the inclusions in metaconcrete can absorb a large amount of wave energy, and reduce the stress on the mortar phase, which greatly improve the resistance of material under dynamic load.

In this work, the impact responses of a decahedral honeycomb sandwich panel structure with internal resonators is studied, and the whole design and optimization process of these resonators are also presented to enhance their attenuation effects.

First, the performances of five kinds of LMs are studied by means of theoretical and numerical analyses. Then, the effects of damping on the impact responses of the panel structure are studied by adding the dampers on the LMs. The impact force and collision energy are significantly attenuated and dissipated through the coupling of metadamping and local resonance. After that, the non-dominated sorting genetic algorithm II (NSGA-II) (Deb et al. 2002) and the artificial tree (AT) algorithm (Li et al. 2017) are applied to optimize the dissipative multi-resonator model (DMM) to further dissipate more impact energy and reduce the structure mass as well as the peak force.

The paper is organized as follows: Section 2 describes the decahedral honeycomb sandwich panel structure. The main content of Section 3 is about the comparative studies of five kinds of LMs. The combination of local resonance property and metadamping characteristic is studied with the DMM in Section 4, and the results show the significant improvement of the impact responses of the panel structure. Then, the optimization analyses of spring stiffness, lumped mass and damping coefficient are carried out in Section 5. Section 6 presents a summary and outlook on the design and manufacture of LMs. Finally, the conclusions are summarized in Section 7.

2. The decahedral honeycomb sandwich panel structure

Fig. 1 shows the decahedral honeycomb sandwich panel structure. From Fig. 1(a), a rigid sphere which mass is 0.21 kg, impacts this structure with initial speed of 64 km/h, and the whole panel structure is constrained at its both ends. The impact force, collision energy and structure deformation are the output responses. The collision analysis is carried out with commercial software LSDYNA. As shown in Fig. 1(b), this panel structure is composed of a cover plate, a base plate and a decahedral honeycomb structure. The thicknesses of the cover plate, the base plate and the decahedral honeycomb structure are all 1.5 mm. The material of all components is aluminum alloy 6060-T6, and its effective stress and effective strain curve is shown in Fig. 1(c). The detailed sizes of the panel structure and one decahedral honeycomb cell

are shown in Fig. 1(d).

(a)

(b)

(c)

Fig. 1. (a) The crash model with the three-dimensional sandwich panel structure. (b) The details of the sandwich panel structure. (c) The effective stress and effective strain curve of aluminum alloy 6060-T6. (d) The sizes of the whole impact model and a single decahedral honeycomb cell.

3. The sandwich panel structure with different internal resonators

3.1. Different kinds of LMs

In this section, the negative mass frequency regions of the physical models of five kinds of LMs are compared to each other, which are the single-resonator, double-resonator, two-resonator, three-resonator and multi-resonator microstructures in Figs. 2(a), (c), (e), (h) and (k).

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

Fig. 2. (a) Single-resonator microstructure. (b) Plot of m_{eff}/m_{st} against ω/ω_2 of single-resonator microstructure. (c) Double-resonator microstructure. (d) Plot of m_{eff}/m_{st} against ω/ω_2 of double-resonator microstructure. (e) Two-resonator microstructure. (f) Surface graph of m_{eff}/m_{st} against ω/ω_2 and ω/ω_3 of two-resonator microstructure in global view. (g) Surface graph of two-resonator microstructure in top view. (h) Three-resonator microstructure. (i) Surface graph of m_{eff}/m_{st} against ω/ω_2 and ω/ω_3 of three-resonator microstructure in global view. (j) Surface graph of three-resonator microstructure in top view. (k) Multi-resonator microstructure. (l) Surface graph of m_{eff}/m_{st} against ω/ω_2 and ω/ω_4 of multi-resonator microstructure in global view. (m) Surface graph of multi-resonator microstructure in top view.

3.1.1 Single-resonator microstructure

The single-resonator microstructure which contains two rigid masses is shown in Fig. 2(a). The outer mass is m_1 which displacement is u_1 . The inner oscillator is m_2 , and its displacement is u_2 . The rigid masses m_2 and m_1 are connected by spring k_2 . Force F is applied to the external mass. Based on Newton's Second Law, the motion equations of m_1 and m_2 are acquired as Eq. (1).

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} d^2 u_1 / dt^2 \\ d^2 u_2 / dt^2 \end{Bmatrix} + \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \quad (1)$$

where t is the time. Supposing that the oscillator movements and the outer force follow the simple harmonic behavior, the non-dimensional effective mass can be derived as Eq. (2).

$$\frac{m_{eff}}{m_{st}} = 1 + \frac{\theta}{1 + \theta} \left[\frac{(\omega/\omega_2)^2}{1 - (\omega/\omega_2)^2} \right] \quad (2)$$

where $\theta = m_2/m_1$ is the ratio of inner mass m_2 to outer mass m_1 . Fig. 2(b) shows the plot of non-dimensional effective mass m_{eff}/m_{st} against non-dimensional frequency ω/ω_2 . From Fig. 2(b), a small band gap appears in the resonance frequency, and the effective mass becomes negative in this narrow frequency range.

3.1.2 Double-resonator microstructure

Fig. 2(c) illustrates the double-resonator microstructure. There are three lumped masses in this microstructure, which are the external mass m_1 , the middle mass m_3 and the internal mass m_2 . These rigid masses are connected by springs k_2 and k_3 . The movements of m_1 , m_3 and m_2 are u_1 , u_3 and u_2 . The motion equations of these oscillators are obtained as Eq. (3).

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} d^2 u_1 / dt^2 \\ d^2 u_2 / dt^2 \\ d^2 u_3 / dt^2 \end{Bmatrix} + \begin{bmatrix} k_3 & 0 & -k_3 \\ 0 & k_2 & -k_2 \\ -k_3 & -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \\ 0 \end{Bmatrix} \quad (3)$$

Based on Eq. (3), the non-dimensional effective mass m_{eff}/m_{st} of this double-resonator microstructure can be calculated as Eq. (4).

$$\frac{m_{eff}}{m_{st}} = \frac{\theta_1}{\theta_1 + \theta_3 + \theta_1 \theta_3} + \frac{\theta_3}{\theta_1 + \theta_3 + \theta_1 \theta_3} \left[\frac{1 + \theta_1 - (\omega/\omega_2)^2}{[1 - (\omega/\omega_2)^2][1 + \delta_1 - (\delta_1 / \theta_1)(\omega/\omega_2)^2] - \delta_1} \right] \quad (4)$$

where the mass ratios are $\theta_1 = m_2/m_3$ and $\theta_3 = m_2/m_1$, the stiffness ratio is $\delta_1 = k_2/k_3$, the local resonance frequency of the inner mass m_2 is $\omega_2 = \sqrt{k_2/m_2}$. Fig. 2(d) shows the plot of m_{eff}/m_{st} against ω/ω_2 . There are two band gaps in Fig. 2(d), and the non-dimensional effective mass is negative in these frequency regions. The width of these two band gaps is larger than that of the single band gap, which means the

double-resonator microstructure can better manipulate the mechanical waves compared with the single-resonator microstructure.

3.1.3 Two-resonator microstructure

Fig. 2(e) illustrates the two-resonator microstructure which also has three lumped masses m_1 , m_2 and m_3 . The displacement of each mass is u_1 , u_2 and u_3 . The springs k_2 and k_3 connect m_1 , m_2 and m_3 . The motion equations of these masses are shown as follows.

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} d^2u_1/dt^2 \\ d^2u_2/dt^2 \\ d^2u_3/dt^2 \end{Bmatrix} + \begin{bmatrix} k_2 + k_3 & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ k_3 & 0 & -k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \\ 0 \end{Bmatrix} \quad (5)$$

Analogous to the single-resonator and double-resonator microstructures, the non-dimensional effective mass m_{eff}/m_{st} of this model is acquired as Eq. (6).

$$\frac{m_{eff}}{m_{st}} = \frac{1}{1+\theta_2+\theta_3} + \frac{1}{1+\theta_2+\theta_3} \left[\theta_2 \frac{1}{1-(\omega/\omega_2)^2} + \theta_3 \frac{1}{1-(\omega/\omega_3)^2} \right] \quad (6)$$

where the mass ratios are $\theta_2 = m_2/m_1$ and $\theta_3 = m_3/m_1$. Surface graphs of m_{eff}/m_{st} against ω/ω_2 and ω/ω_3 in global and top views are shown in Figs. 2(f) and (g). Two orthogonal band gaps appear in these surface plots. When ω/ω_2 and ω/ω_3 approach 1, two band gaps are generated on the ω/ω_2 and ω/ω_3 axes, respectively. Differ from the double-resonator microstructure, there is no coupling between these two band gaps, which is convenient for us to design the two band gaps separately to mitigate the critical spectrum areas.

3.1.4 Three-resonator microstructure

The three-resonator microstructure is illustrated in Fig. 2(h). There are four rigid masses m_1 , m_2 , m_3 and m_4 in this microstructure, and their displacements are u_1 , u_2 , u_3 and u_4 . The spring k_2 connects lumped masses m_2 and m_4 , the spring k_3 connects resonators m_3 and m_4 , and the spring k_4 connects oscillators m_1 and m_4 . The motion equations of these resonators can be written as follows.

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{Bmatrix} d^2 u_1 / dt^2 \\ d^2 u_2 / dt^2 \\ d^2 u_3 / dt^2 \\ d^2 u_4 / dt^2 \end{Bmatrix} + \begin{bmatrix} k_4 & 0 & 0 & -k_4 \\ 0 & k_2 & 0 & -k_4 \\ 0 & 0 & k_3 & -k_3 \\ -k_4 & -k_2 & -k_3 & k_2 + k_3 + k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (7)$$

Then, the effective mass m_{eff} of this three-resonator microstructure can be obtained as Eq. (8).

$$m_{eff} = m_1 + m_4 H \quad (8)$$

where H represents $\frac{1 + \frac{\theta_2}{1 - (\omega/\omega_2)^2} + \frac{\theta_3}{1 - (\omega/\omega_3)^2}}{1 - \frac{\delta_2(\omega/\omega_2)^2}{2\theta_2} - \frac{\delta_3(\omega/\omega_3)^2}{2\theta_3} - \frac{\delta_2(\omega/\omega_2)^2}{1 - (\omega/\omega_2)^2} - \frac{\delta_3(\omega/\omega_3)^2}{1 - (\omega/\omega_3)^2}}$. The

stiffness ratios are $\delta_2 = k_2/k_4$ and $\delta_3 = k_3/k_4$. The mass ratios are $\theta_2 = m_2/m_4$ and $\theta_3 = m_3/m_4$. Finally, the non-dimensional effective mass m_{eff}/m_{st} of this new microstructure can be acquired as Eq. (9).

$$\begin{aligned} \frac{m_{eff}}{m_{st}} &= \frac{1}{1 + \theta_2 \gamma_4 + \theta_3 \gamma_4 + \gamma_4} \\ &+ \frac{\gamma_4}{1 + \theta_2 \gamma_4 + \theta_3 \gamma_4 + \gamma_4} \frac{1 + \frac{\theta_2}{1 - (\omega/\omega_2)^2} + \frac{\theta_3}{1 - (\omega/\omega_3)^2}}{1 - \frac{\delta_2(\omega/\omega_2)^2}{2\theta_2} - \frac{\delta_3(\omega/\omega_3)^2}{2\theta_3} - \frac{\delta_2(\omega/\omega_2)^2}{1 - (\omega/\omega_2)^2} - \frac{\delta_3(\omega/\omega_3)^2}{1 - (\omega/\omega_3)^2}} \end{aligned} \quad (9)$$

where $\gamma_4 = m_4/m_1$ is the ratio of middle mass to outer mass.

Figs. 2(i) and (j) show the surface graphs of m_{eff}/m_{st} against ω/ω_2 and ω/ω_3 in global and top views. Three band gaps appear in these surface graphs, and these band gaps are symmetric along the center line of ω/ω_2 and ω/ω_3 axes. These large negative effective mass frequency regions imply its great performance in manipulate mechanical waves.

3.1.5 Multi-resonator microstructure

As shown in Fig. 2(k), the multi-resonator microstructure is composed of five rigid masses m_1, m_2, m_3, m_4 and m_5 . The movements of these oscillators are u_1, u_2, u_3, u_4 and u_5 , respectively, and the spring which connects these resonators are k_1, k_2, k_3 and k_4 . Therefore, the control equation of the multi-resonator microstructure is

summarized as follows.

$$\begin{aligned}
& \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix} \begin{Bmatrix} d^2 u_1 / dt^2 \\ d^2 u_2 / dt^2 \\ d^2 u_3 / dt^2 \\ d^2 u_4 / dt^2 \\ d^2 u_5 / dt^2 \end{Bmatrix} + \\
& \begin{bmatrix} k_3 + k_5 & 0 & -k_3 & 0 & -k_5 \\ 0 & k_2 & -k_2 & 0 & 0 \\ -k_3 & -k_2 & k_2 + k_3 & 0 & 0 \\ 0 & 0 & 0 & k_4 & -k_4 \\ -k_5 & 0 & 0 & -k_4 & k_4 + k_5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (10)
\end{aligned}$$

Then, the non-dimensional effective mass m_{eff}/m_{st} of this multi-resonator microstructure can be obtained as Eq. (11).

$$\begin{aligned}
\frac{m_{eff}}{m_{st}} &= \frac{1}{1 + (\theta_2 + 1)\gamma_3 + (1 + \theta_4)\gamma_5} \\
&+ \frac{\gamma_3}{1 + (\theta_2 + 1)\gamma_3 + (1 + \theta_4)\gamma_5} \frac{((\omega/\omega_2)^2 - \theta_2 - 1)\theta_2}{\delta_2(\omega/\omega_2)^2(\theta_2 - (\omega/\omega_2)^2 + 1) + \theta_2((\omega/\omega_2)^2 - 1)} \\
&+ \frac{\gamma_5}{1 + (\theta_2 + 1)\gamma_3 + (1 + \theta_4)\gamma_5} \frac{((\omega/\omega_4)^2 - \theta_4 - 1)\theta_4}{\delta_4(\omega/\omega_4)^2(\theta_4 - (\omega/\omega_4)^2 + 1) + \theta_4((\omega/\omega_4)^2 - 1)} \quad (11)
\end{aligned}$$

where the mass ratios are $\theta_2 = m_2/m_3$, $\gamma_3 = m_3/m_1$, $\theta_4 = m_4/m_5$ and $\gamma_5 = m_5/m_1$, the stiffness ratios are $\delta_2 = k_2/k_3$ and $\delta_4 = k_4/k_5$. Figs. 2(l) and (m) are the surface graphs of m_{eff}/m_{st} against ω/ω_2 and ω/ω_4 in global and top views. There are four band gaps in Figs. 2(l) and (m), two of these band gaps appear on the ω/ω_2 axis and the other two appear on the ω/ω_4 axis. Furthermore, these multiple band gaps mean wide negative mass frequency regions and great design freedom for spectrum attenuation.

3.2. Impact analyses of different resonator models

As shown in Fig. 3, an oscillator is applied in one decahedral honeycomb cell, and the combination of one oscillator and one decahedral honeycomb cell constitutes a single-resonator microstructure. The decahedral honeycomb cell represents the outer mass m_1 of the microstructure. Based on the same principle, the other four types of oscillators are also applied in this sandwich panel structure, respectively. With this combination of the resonators and the decahedral honeycomb cells, this sandwich

panel structure has the ability to mitigate the impact force. Then, the attenuation effects of the panel structure with these five kinds of resonators are compared with each other by studying their impact results. The oscillator masses of all LMs are 0.005 kg, and the spring stiffness of each microstructure is shown in Table 1.

Fig. 3. The panel structure with single-resonator.

Table 1. The spring stiffness of these five kinds of LMs

	Spring stiffness (N/mm)
Single-resonator	$k_2 = 7.9$
Double-resonator	$k_2 = 17.7, k_3 = 7.9$
Two-resonator	$k_2 = 17.7, k_3 = 7.9$
Three-resonator	$k_2 = 31.6, k_3 = 17.7, k_4 = 7.9$
Multi-resonator	$k_2 = 31.6, k_3 = 1.8, k_4 = 17.7, k_5 = 7.9$

The results of impact forces of these models are shown in Figs. 4, 5 and Table 2. The original model is the sandwich structure without any oscillator, and its peak impact force is 23.9 kN. The peak forces of the sandwich panel structure with single-resonator, double-resonator, two-resonator, three-resonator and multi-resonator cores are 22.1 kN, 19.9 kN, 19.2 kN, 17.7 kN and 15.3 kN, respectively. Obviously, the impact force is significantly attenuated with these LMs. Furthermore, with the same unit cell number, the attenuation effect of the multi-resonator is the best among these models.

From Figs. 4(a) and (b), the impact force, kinetic energy and internal energy curves of original model keep oscillating with time. The initial kinetic energy and

initial internal energy are 34.16 J and 0 J. These imply the absorption efficiency of impact energy is low. In addition, Fig. 4 (c) shows the deformation nephogram of the sandwich panel structure at different time. It can be seen that when $t = 2$ ms, the deformation is mainly concentrated in the middle area of the structure, and the maximum deformation is 4.64 mm. When $t = 5$ ms, the deformation zone is enlarged, and its peak deformation is 6.58 mm. Furthermore, when the time is increased to 10ms, 15ms or 20ms, the deformation region of the structure is still very large, and the maximum deformations are 4.51 mm, 6.05 mm and 6.20, respectively. Therefore, the deformation region of the structure is large, and the deformation does not decrease with time.

From Fig. 5, for these resonator models, their amplitudes of impact force, kinetic energy and internal energy curves are reduced compared with the original model, and the decline of the MM is the most pronounced. These mean that much impact energy is stored in the MM. Therefore, based on the best performances of impact force mitigation and impact energy storage, the multi-resonator is selected for next study.

(a)

(b)

(c)

Fig. 4. The impact results of the original model of (a) impact force curve, (b) kinetic (Kin) energy and internal (Int) energy curves, (c) deformation nephogram.

(a)

(b)

(c)

(d)

(e)

Fig. 5. Impact force and impact energy curves of the (a) single-resonator, (b) double-resonator, (c) two-resonator, (d) three-resonator, and (e) multi-resonator models.

Table 2. The impact results these five kinds of resonator models

	Peak impact force (kN)
Original	23.9
Single-resonator	22.1
Double-resonator	19.9
Two-resonator	19.9
Three-resonator	17.7
Multi-resonator	15.3

4. Sandwich panel structure with dissipative multi-resonators

4.1 Dissipative multi-resonator microstructure

In this section, the effect of damping characteristic on the performance of MM is studied, and the damper is applied on the MM. Fig. 6(a) illustrates the dissipative multi-resonator microstructure. This structure is composed of the multi-resonator microstructure and four dashpots with damping coefficient c_2 , c_3 , c_4 and c_5 in parallel to the spring k_2 , k_3 , k_4 and k_5 . The control equations of this microstructure are obtained as Eq. (12).

$$\begin{aligned}
m_2 \frac{d^2 u_2}{dt^2} &= k_2 (u_3 - u_2) + c_2 \left(\frac{du_3}{dt} - \frac{du_2}{dt} \right) \\
m_3 \frac{d^2 u_3}{dt^2} &= k_2 (u_2 - u_3) + k_3 (u_1 - u_3) + c_2 \left(\frac{du_2}{dt} - \frac{du_3}{dt} \right) + c_3 \left(\frac{du_1}{dt} - \frac{du_3}{dt} \right) \\
m_1 \frac{d^2 u_1}{dt^2} &= F + k_3 (u_3 - u_1) + k_5 (u_5 - u_1) + c_3 \left(\frac{du_3}{dt} - \frac{du_1}{dt} \right) + c_5 \left(\frac{du_5}{dt} - \frac{du_1}{dt} \right) \\
m_4 \frac{d^2 u_4}{dt^2} &= k_4 (u_5 - u_4) + c_4 \left(\frac{du_5}{dt} - \frac{du_4}{dt} \right) \\
m_5 \frac{d^2 u_5}{dt^2} &= k_5 (u_1 - u_5) + k_4 (u_4 - u_5) + c_5 \left(\frac{du_1}{dt} - \frac{du_5}{dt} \right) + c_4 \left(\frac{du_4}{dt} - \frac{du_5}{dt} \right)
\end{aligned} \tag{12}$$

Based on Eq. (12), the following Eq. (13) can be calculated.

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \\ A_{41} & A_{42} & A_{43} \\ A_{51} & A_{52} & A_{53} \end{bmatrix} \begin{Bmatrix} \omega^2 \\ \omega \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (13)$$

in which

$$A_{11} = m_2 \hat{u}_2, \quad A_{21} = m_3 \hat{u}_3, \quad A_{31} = m_1 \hat{u}_1, \quad A_{41} = m_4 \hat{u}_4, \quad A_{51} = m_5 \hat{u}_5, \quad A_{12} = -ic_2 (\hat{u}_3 - \hat{u}_2),$$

$$A_{22} = -i[c_3 (\hat{u}_1 - \hat{u}_3) + c_2 (\hat{u}_2 - \hat{u}_3)], \quad A_{32} = -i[c_3 (\hat{u}_3 - \hat{u}_1) + c_5 (\hat{u}_5 - \hat{u}_1)], \quad A_{42} = -ic_4 (\hat{u}_5 - \hat{u}_4),$$

$$A_{52} = -i[c_4 (\hat{u}_4 - \hat{u}_5) + c_5 (\hat{u}_1 - \hat{u}_5)], \quad A_{13} = k_2 (\hat{u}_3 - \hat{u}_2), \quad A_{23} = k_3 (\hat{u}_1 - \hat{u}_3) + k_2 (\hat{u}_2 - \hat{u}_3),$$

$$A_{33} = k_3 (\hat{u}_3 - \hat{u}_1) + k_5 (\hat{u}_5 - \hat{u}_1) + \hat{F}, \quad A_{43} = k_4 (\hat{u}_5 - \hat{u}_4), \quad A_{53} = k_4 (\hat{u}_4 - \hat{u}_5) + k_5 (\hat{u}_1 - \hat{u}_5),$$

$\hat{u}_i$ is the amplitude of u_i and $\hat{F}$ is the amplitude of F .

In order to reveal the working mechanism, the dissipative multi-resonator microstructure is equivalent to two effective media, the effective mass and the effective metadamping. Based on Newton's second law, the following equation

$$F = m_{eff} \frac{d^2 u_1}{dt^2} + c_{eff} \frac{du_1}{dt} \quad \text{must be satisfied for this microstructure, where } c_{eff} \text{ is the}$$

effective metadamping coefficient and m_{eff} is the effective mass. Large c_{eff} means that the damping effect is obvious, and the impact energy can be effectively dissipated.

Then, the following Eq. (14) can be derived.

$$\hat{F} = -\hat{u}_1 (m_{eff} \omega^2 + ic_{eff} \omega) \quad (14)$$

Fig. 6. (a) The dissipative multi-resonator microstructure. (b) Surface graph of m_{eff}/m_{st} against ω/ω_2 and ω/ω_4 in global view. (c) Surface plot of m_{eff}/m_{st} against ω/ω_2 and ω/ω_4 in left view. (d) Surface graph of c_{eff} against ω/ω_2 and ω/ω_4 .

Figs. 6(b), (c) and (d) are the surface graphs of m_{eff}/m_{st} and c_{eff} against ω/ω_2 and ω/ω_4 . There are four perpendicular band gaps for m_{eff}/m_{st} in Fig. 6(b) which is the same as the multi-resonator microstructure. Fig. 6(c) presents the left view of m_{eff}/m_{st} surface graph. It is clear that there are two band gaps on the ω/ω_2 axis. Fig. 6(d) shows four band gaps for the metadamping surface graph, and the value of effective metadamping c_{eff} is always positive. Its magnitude is significantly affected by the resonant frequencies. Near first and second resonant frequencies on the ω/ω_2 and ω/ω_4 axes, the value of c_{eff} is significantly improved. Therefore, the mechanical wave can be obviously dissipated at the resonance frequencies with the DMM.

4.2 Impact analyses of dissipative multi-resonator model (DDM)

The sandwich panel structure with dissipative multi-resonator is shown in Fig. 7. In this model, the damping coefficients are $c_1 = 1.0$ Nm/s, $c_2 = 5.0$ Nm/s, $c_3 = 7.0$ Nm/s

and $c_4 = 2.0$ Nm/s. The impact results are shown in Fig. 8 and Table 3. In Fig. 8(a) and Table 3, the peak impact force of the DMM is 14.01 kN, which is obviously attenuated compared with the original and MM. Furthermore, the kinetic energy is significantly reduced and the internal energy is obviously raised from Fig. 8(b). The oscillation frequency and amplitude of impact force and impact energy of DMM are reduced quickly over time compared with the original and MM. The values of internal energy at 20 ms (INT20) and kinetic energy at 20 ms (KIN20) are 30.67 J and 3.08 J. It is obvious that the impact energy is significantly dissipated. All of these results obviously demonstrate the attenuation and dissipation effects of DMM on the stress waves. The combination of viscoelastic and oscillators improves the performances of the panel structure.

Fig. 7. The panel structure with dissipative multi-resonators.

Fig. 8. Plots of (a) the impact force curve, (b) the internal and kinetic curves of DMM.

Table 3. The impact results of DMM.

	Peak impact force (kN)	INT20 (J)	KIN20 (J)
DMM	14.01	3.08	30.65

5. Optimize the DMM

It is clear that the spring stiffness, oscillator masses and damping coefficients significantly affect the performance of DMM. Especially for the oscillator mass, heavier resonator can absorb and store more impact energy. However, it is worth noting that heavy oscillator increase the structure mass and restrict the application of DMM. Therefore, the parameters of DMM should be designed to improve the structural response and reduce the structure mass. Then, several optimization problems are proposed, and the optimization goals include the maximization of force attenuation and energy dissipation as well as the minimization of structure mass and unit cell number. The parameters of spring stiffness, oscillator mass and damping coefficient are set as the design variables. The entire optimization process is split into two steps to reduce the design variables in one optimization process. In addition, due to the symmetry of the structure, one quarter of the entire plate structure is taken as the design area. Among these parameters, the spring stiffness and the resonator mass should be optimized together, as they affect the resonance frequencies and band gaps together. First, regarding the parameters of spring stiffness and resonator mass in all unit cells of the plate structure as the design variables, and the damping coefficients remain constant. The NSGA-II is applied as the optimization theory. The optimization goals are to maximize the load attenuation and minimize the structure mass. After this optimization process, the light unit cells are removed to reduce the number of cells. Finally, with these designed unit cells, the damping coefficients are optimized. In this process, the goal is to minimize the peak impact force, and AT algorithm is applied to deal with this single-object problem.

5.1. Optimize the spring stiffness and the oscillator mass

The optimization analysis is carried out with the spring stiffness and the resonator mass. The design spaces of the variables are shown in Table 4. There are twenty-four groups of unit cells in this model, and each group has four stiffness and four mass parameters. In order to further simplify the optimization process, the

relationship of the spring stiffness and vibrator mass are defined as $k_3 = 3.3k_2 = 2.4k_4 = 0.7k_5$ and $m_2 = m_3 = m_4 = m_5$, and the total design variables are reduced to forty-eight. The mathematical model of the multi-objective optimization problem is written as Eq. (15).

$$\begin{cases} \text{Min } F_{peak}, M \\ \text{s.t.} \\ 0.0002 \text{ N/mm} \leq k_3 \leq 0.1 \text{ N/mm} \\ 0.0 \text{ kg} \leq m_2 \leq 0.008 \text{ kg} \\ k_3 = 3.3k_2 = 2.4k_4 = 0.7k_5 \\ m_2 = m_3 = m_4 = m_5 \end{cases} \quad (15)$$

where F_{peak} is the peak impact force, and M is the mass of DMM. The Pareto frontier of this problem is illustrated in Fig. 9. As shown in Fig. 9, fifty feasible solutions are exhibited for us to select.

Table 4. The design space of spring stiffness k_3 and resonator mass m_2 .

Variables	Lower	Upper
k_3 (N/mm)	0.0002	0.1
m_2 (kg)	0.0	0.008

Fig. 9. The Pareto frontier solutions of the WDMM.

In this work, a small structure mass is preferred. Then, the impact responses of the optimal solution are shown in Fig. 10(a) and Table 5, and its peak force, DMM mass, INT20 and KIN20 are 10.42 kN, 5.59 kg, 30.59 J and 3.11 J, respectively.

Furthermore, this optimal model is called the whole designed DMM (WDMM) for the sake of convenience. It is obvious that the impact force of the WDMM is obviously attenuated compared with the original and DMM. Meanwhile, the values of INT20 and KIN20 of WDMM are similar to those of DMM. Therefore, the impact energy is also significantly dissipated. In addition, the mass of all resonators in WDMM are reduced from 12.16 kg to 5.59 kg. These results demonstrate that the optimal parameters of the DMM can effectively improve the performances of the panel structure. The distributions of m_2 and k_3 of each group in WDMM are shown in Figs. 11(a), (b) and Tables 6, 7. In Figs. 11(a) and (b), the diameter of the ball and the size of the spring are linear with the values of m_2 and k_3 . Therefore, a big ball means a heavy resonator, and a big ring implies the high stiffness of the spring. The mass and spring stiffness distributions of all cells can be roughly seen by using these figures. In Fig. 11 (a), it can be seen that the resonator masses in the impact zone are bigger than those of other regions. This is because the attenuation and dissipation of stress waves requires heavy resonators.

Table 5. The crash results of all models.

	Peak impact force (kN)	INT20 (J)	KIN20 (J)	Mass (kg)	Unit cell number
Original	23.9	—	—	—	—
DMM	14.01	3.08	30.65	12.16	608
WDMM	10.42	3.11	30.59	5.59	608
RDMM	10.84	3.10	30.60	5.37	496

Table 6. The values of designed m_2 in all groups.

	m_2 (kg)	Group	m_2 (kg)	Group	m_2 (kg)
Group 1	0.003118	Group 3	0.000756	Group 5	0.001162
Group 2	0.000922	Group 4	0.000267	Group 6	0.001075
Group 7	0.002752	Group 9	0.002238	Group 11	0.002139
Group 8	0.001731	Group 10	0.003148	Group 12	0.001703
Group 13	0.004082	Group 15	0.003313	Group 17	0.000422
Group 14	0.000484	Group 16	0.000961	Group 18	0.001130
Group 19	0.006745	Group 21	0.003135	Group 23	0.003883
Group 20	0.000856	Group 22	0.003902	Group 24	0.007492

Table 7. The values of designed k_3 in all groups.

	k_3 (kN/mm)	Group	k_3 (kN/mm)	Group	k_3 (kN/mm)
Group 1	0.035765	Group 3	0.036333	Group 5	0.090197
Group 2	0.095675	Group 4	0.043323	Group 6	0.077760
Group 7	0.065625	Group 9	0.054805	Group 11	0.063241
Group 8	0.012946	Group 10	0.056479	Group 12	0.043216
Group 13	0.092746	Group 15	0.086920	Group 17	0.044580
Group 14	0.037566	Group 16	0.081782	Group 18	0.001069
Group 19	0.075173	Group 21	0.010816	Group 23	0.093135
Group 20	0.084223	Group 22	0.001989	Group 24	0.024374

From Fig. 11(a) and Table 6, there are many light unit cells, and these cells have less influence on the performances of WDM. In addition, for practical engineering problems, more unit cells mean more design and manufacturing costs. Therefore, these unit cells with oscillator masses less than 0.0008 kg are deleted, and the WDM with less unit cells is named the reduced DMM (RDMM). Finally, the number of cells of the RDMM decreases from 608 to 496, and the impact results of the RDMM are shown in Fig. 10(b) and Table 5. The peak force, structure mass, INT20 and KIN20 of the RDMM are 10.84 kN, 5.37 kg, 30.60 J and 3.10 J, respectively. Compared with the WDM, the peak force of the RDMM is slightly increased, while its structure mass and unit cell number are significantly reduced. Figs. 11(c) and (d) show the distributions of m_2 and k_3 of all cells. It is clear that these light oscillators as well as their corresponding springs on the both side and the middle regions of the structure are removed.

(a)

(b)

Fig. 10. Impact force and impact energy curves of (a) the WDMM, (b) the RDMM.

(a)

(b)

(c)

Fig. 11. The distributions of (a) resonator mass and (b) spring stiffness of the WDMM. The distributions of (c) resonator mass and (d) spring stiffness of the RDMM.

5.2. Optimize the damping coefficients

The optimization analysis is carried out with the damping coefficients to further enhance its attenuation and dissipation effects. This optimization model contains six groups of cells and twenty-four design variables. The goal is to minimize the impact force, and the optimization theory is AT. The range of c_2 is from 1 Nm/s to 80 Nm/s. The mathematical model of this single-objective problem is shown as follows.

$$\begin{cases} \text{Min } F_{peak} \\ \text{s.t.} \\ 1 \text{ Nm/s} \leq c_2 \leq 80 \text{ Nm/s} \\ 1 \text{ Nm/s} \leq c_3 \leq 80 \text{ Nm/s} \\ 1 \text{ Nm/s} \leq c_4 \leq 80 \text{ Nm/s} \\ 1 \text{ Nm/s} \leq c_5 \leq 80 \text{ Nm/s} \end{cases} \quad (16)$$

The optimal model is called the designed damping RDMM (DDMM), and the optimum results are shown in Fig. 12 and Table 8. From Fig. 12(a), (b) and Table 8, the peak impact force, INT20 and KIN20 of DDMM are 9.10 kN, 2.23 J and 31.44 J, respectively. The optimal damping coefficients of each group are shown in Table 9. It is obvious that the values of peak impact force and KIN20 of DDMM are the smaller than those of RDMM, and its value of INT20 is the larger than that of RDMM. Therefore, the results of the DDMM are significantly better than those of the RDMM, and the mitigation of peak force and the dissipation of impact energy are very obvious. Fig. 12(c) shows the displacement nephogram of the panel structure at different time.

It is clear that the deformation is mainly concentrated in the impact area of the structure during the entire collision process. In other zone, the structure deformation is very small. Compared with the original model, the deformation of the DDMM in other regions is significantly reduced. Therefore, the structures outside the collision region can be effectively protected with DDMM.

The greatly improved performance of the DDMM in dynamic load mitigation and impact energy dissipation indicates the wide application prospects of the combination of honeycomb sandwich structure and resonators, and this part of work can be further extend to other areas. For example, this combined structure can be applied to alleviate the impact force in vehicle collision, isolate the impact load and mechanical vibration suffered by power battery and relieve damage caused by explosion dynamic loading.

(a)

(b)

Fig. 12. (a) Impact force curve, (b) energy curves, (c) deformation nephogram of the DDMM.

Table 8. The impact results of the DDMM

	Peak impact force (kN)	INT20 (J)	KIN20 (J)	Mass (kg)	Unit cell number
DDMM	9.10	31.44	2.13	5.37	496

Table 9. The designed damping coefficients of each group

	c_1 (Nm/s)	c_2 (Nm/s)	c_3 (Nm/s)	c_4 (Nm/s)
Group 1	0.02215	0.04018	0.03982	0.00200
Group 2	0.00700	0.02954	0.03840	0.03031
Group 3	0.00584	0.02567	0.05061	0.00182
Group 4	0.00254	0.04165	0.05585	0.00387
Group 5	0.02452	0.01196	0.01296	0.01964
Group 6	0.05207	0.05837	0.05006	0.05971

6. Discussion of the practical design and manufacturing of LMs

This section presents a summary and outlook of the practical design and manufacturing of LMs. LMs have a variety of structural forms, and previous studies

have already given advices on the design of LMs. One kind of LMs was fabricated with the centimeter-sized lead balls and the resin coating of a thickness of 2.5 mm (Ping et al. 2000). The coatings and spheres are arranged in a simple cubic crystal with a lattice constant of 1.55 cm. Two types of LMs were provided by (Manimala et al. 2014). One is the coil spring oscillator and the other is the improved flat spring oscillator. The local resonance frequency of the coil spring resonator varies greatly, and the flat spring oscillator uses a thin steel plate designed by finite element simulation to provide a more accurate stiffness for the required local resonant frequency. A spring mass system was proposed by (Khan, Li, and Tan 2018). In this system, carbon steel is used as the material of inner oscillator, and the lightweight aluminum alloy structure is used for external mass. Coil springs are used to connect internal and external masses, and adhesives with epoxy and hardener connect the springs to internal and external masses. For the current work of (Barnhart et al. 2019), the dissipative double resonator microstructure which is embedded in an epoxy resin matrix by a double layered spherical inclusion was fabricated. The experiment results show this microstructure can achieve the mitigation of broadband elastic wave.

The conceptual design to combine the honeycomb structure and resonators to mitigate the impact load and dissipate the impact energy is presented in this work. Based on the results of theoretical analyses and numerical calculations, it is seen that the impact responses of the sandwich panel structure are greatly improved. It is clear that the combination of resonators and honeycomb sandwich structures shows the great potential for structural collision problems. In addition, our work is in its infancy, and there is still much room to explore, especially in the actual design, manufacturing and testing of LMs.

7. Conclusion

In this paper, the extraordinary attenuation effects of a decahedral honeycomb sandwich panel structure with internal resonators are studied. Combined with the matrix structure, the resonators present the bizarre property of negative effective mass, and the combination of damping characteristic and local resonance of resonator

exhibits the phenomenon of metadamping. The theoretical and numerical analyses show that these novel properties can effectively attenuate the impact load and dissipate the impact energy. In order to further enhance the structure responses, the spring stiffness, lumped mass and damping coefficient of internal resonators are optimized based on optimization algorithms. The optimized sandwich structure presents extraordinary load attenuation and energy absorption with a relatively small structure mass. The combination of resonators and honeycomb sandwich structure shows great potential application in many areas.

Disclosure statement

No potential conflict of interest was reported by the authors.

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