Flocking of multi-agent systems with unknown nonlinear dynamics and heterogeneous virtual leader

Tingruo Yan, Xu Xu*, Zongying Li, and Eric Li

Abstract: This paper investigates the flocking control of multi-agent systems with unknown nonlinear dynamics while the virtual leader information is heterogeneous. The uncertain nonlinearity in the virtual leader information is considered, and the weaker constraint on the velocity information measurements is assumed. In addition, a bounded assumption on the unknown nonlinear dynamics is also considered. It is weaker than the Lipschitz condition adopted in the most flocking control methods. To avoid fragmentation, we construct a new potential function based on the penalty idea when the initial network is disconnected. A dynamical control law including a adjust parameter is designed to achieve the stable flocking. It is proven that the velocities of all agents approach to consensus and no collision happens between the mobile agents. Finally, several simulations verify the effectiveness of the new design, and indicate that the proposed method has high convergence and the broader applicability in practical applications with more stringent restrictions.

Keywords: flocking, connectivity preserving, potential function, uncertain nonlinearity, unknown dynamics.

1. INTRODUCTION

Flocking is a typical collective behavior that the mobile agents use simple rules to organize into a coordinated motion. In the past decades, the flocking control of multiple agents has become a major concern in many fields of science and engineering. Researchers paid much more efforts to understand how the crowds of people, swarming of bacteria and a group of birds can cluster in formations [1-3]. In the real-world problems, learning the mechanism of cooperative motions in biological groups may be helpful to develop many artificial autonomous problems such as crowd evacuation, transport management, and formation control of the intelligent vehicles. Recently, many flocking control approaches on how to use the local information to ensure the global behaviors have been proposed [4-6].

Reynolds first defined the flocking model with three basic rules: Separation, Alignment and Cohesion [7]. After that, many flocking models had been proposed. A potential function was proposed in [8], and its gradient can be acted as the repulsive force or attractive force to satisfy the above rules. To avoid the nonsmooth analysis, Olfati [9] constructed a smooth potential function that guarantees the continuity of energy at the time of switching topology. Then, the results of [10-12] focused on problems of the multi-agents with fixed and switching topologies. The concept of the infinite potential function was employed in [13], which designs the infinite force to avoid splitting when two agents are about to leave their sensing ranges. Su [14] addressed a connectivity-preserving flocking algorithm without velocity measurements, where a delayed edge-adding method was presented to ensure the finite energy at the time of switching topology. Under the assumption of the joint connection, Su and Lin [15] proposed a connectivity enhancing coordinated control with the piecewise potential energy function. Gao and Xu [16] considered a more flexible distributed pinning topology control to avoid the fragmentation of the whole network.

Most existing works on the flocking considered the constraints of the nonlinear dynamics satisfying the Lipschitz condition [3,17-18]. However, the control protocols may fail to work when the systems include the more complicated unknown nonlinear dynamics [19-20].

In addition, there is a common assumption on the flocking with a virtual leader that some agents can receive the accurate velocity measurements of the virtual

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leader [3,15-18,21-22]. However, in the real-world problems, accurate velocity information from the virtual leader may be difficult to get because of the external disturbances or the measurement errors coming from the speed sensors or the virtual leader itself. For example, Unmanned Aerial Vehicles (UAVs) may be affected by the electromagnetic signal from the enemy and thus the information or operational instructions from the ground may be disturbed. Thus, a more reasonable assumption is that the velocities of some agents may be nonlinear with external disturbances [23-25]. In [25], Wen et al. proposed a pinning flocking algorithm by considering the nonlinear relative velocities between the agents and the virtual leader. Their results assumed that the nonlinear velocity must keep the same direction with desired direction in order to achieve the flocking. However, such assumption cannot be satisfied when the information of the virtual leader exists uncertain nonlinearity and become heterogeneous.

On the other hand, in the flocking problems, some works strongly rely on the assumption that the underlying network is connected at all time [8,13,25-26]. Some researchers employed the infinite potential energy function at both end to ensure the connectivity when the initial network is connected [3,14]. However, in the real-world problems, the network topology is generally not fixed or connected all the time but time-varying and disconnected [16,27-28]. For example, UAVs sometimes have to be disconnected to avoid obstacles, and thus the network encounters disconnectivity in the neighbor graph. Therefore, it should consider the cases that the initial topology of the agent network is disconnected. In this case, the coordinated control would become more challenging.

Motivated by the above analysis, we develop a flocking algorithm with the unknown nonlinear dynamics and a heterogeneous virtual leader. A novel flocking control with a dynamical parameter is proposed to guarantee the flocking of the whole group. The advantages of this algorithm compared with the existing results are as follows. 1) the uncertainty of the velocity measurements of the virtual leader is only bounded; 2) The unknown nonlinear dynamics of the agents do not require to satisfy the Lipschitz condition, and it is only needed to be bounded; 3) A new potential function including a penalty term is constructed to guarantee the connectivity regardless of the connectivity of the initial network.

The outline of this paper is organized as follows: Section 2 gives the preliminaries of the flocking problems. In Section 3, the idea of the proposed algorithm and main results are presented. The theoretical analysis of the stable flocking is investigated in Section 4. Numerical experiments are shown in Section 5 to demonstrate the stability and the higher efficiency of the proposed method. Finally, Section 6 concludes this work.

2. PRELIMINARIES

The agent network can be represented by an undirected graph $G(t)=\{v,e(t)\}$, whose vertices $v=\{1,2,\cdots,n\}$ represent the agents in the network, and a time-varying set of edges $e(t)=\{(i,j)\in v\times v\}$ represents neighboring relations among agents at time t. Vertex i and vertex j are said to be "adjacent" if $(i,j)\in e$. If the neighboring set of any agents changes dynamically with time, then the system has dynamic topological structure; otherwise the system has a fixed topology. An alternating sequence of distinct vertices and edges in the graph is called a "path", and a graph $G(t)$ is "connected" if there is a path between any pair of distinct nodes.

Matrix $D=\{d_{ik}\}\in \mathrm{R}^{n\times|e|}$ represents the incidence matrix associated with graph G, where $d_{ik}=1$ if the edge e_k enters node i, $d_{ik}=-1$ if the edge e_k leaves node i, and $d_{ik}=0$ otherwise. The adjacent matrix $A(G)=\{a_{ij}(t)\}_{i,j=1}^{n}$ associated with graph G is defined as

$$a_{ij}(t)=\begin{cases}1, & if\ \ (i,j)\in e\\ 0, & else\end{cases} \tag{1}$$

The degree matrix is expressed as $M(G)$, where the i^{th} diagonal element is the degree of the node i. Define the Laplacian matrix as $L(G)=M(G)-A(G)$, which is a symmetric and positive semi-definite matrix and $L=DD^{\mathrm{T}}$. The eigenvalues of L can be written as $\lambda_1(L)\le\lambda_2(L)\le\cdots\le\lambda_n(L)$. It is obvious that $\lambda_1=0$ and its corresponding eigenvector is $[1,1,...,1]^{\mathrm{T}}\in\mathrm{R}^n$, and then G is connected if and only if $\lambda_2>0$. The corresponding m-dimensional graph Laplacian is defined as $\hat{L}(t)=L(t)\otimes I_m$, where I_m is the identity matrix of order m, and $\otimes$ stands for the Kronecker product.

3. PROBLEMS FORMULATION AND MAIN RESULTS

We consider n agents moving in a m-dimensional Euclidean space. The dynamic equation for each agent is described by

$$\begin{cases}\dot{q}_i=p_i,\\ \dot{p}_i=u_i+f(p_i), \quad i=1,2,\cdots,n\end{cases} \tag{2}$$

where $q_i,p_i,u_i\in\mathrm{R}^m$ are the position, velocity and control input of agent i, respectively. $f(p_i)\in\mathrm{R}^m$ is the unknown dynamic vector of agent i.

Most existing works required that the dynamics $f(\cdot)$ satisfy the Lipschitz condition [3,17-18]. However, the motion dynamics of real agents in the real-world problems often include the unknown nonlinearity or unknown disturbances which cannot satisfy Lipschitz conditions. In this paper, we consider the unknown dynamics with the following bounded assumption:

Assumption 1: The unknown dynamics $f(\cdot)$ in (2) satisfies

$$\|f(x)-f(y)\|_2\le\kappa, \quad \forall x,y\in\mathrm{R}^m, \tag{3}$$

with the positive constant κ.

The assumption 1 is not equivalent to the Lipschitz condition. In a strictly mathematical way, the Lipschitz condition shows that the function needs to be continuous and has finite derivatives. The unknown dynamics $f(\cdot)$ in the assumption 1 do not require continuity and finite derivatives, and only needs to be bounded.

Suppose all agents have the same sensing radius $r > 0$. The initial edges are generated by $e(t_0) = \{(i, j) : \| q_i(t_0) - q_j(t_0) \| < r, i, j \in v\}$, and the new edges generated during the evolution are based on the hysteresis method [14]. Using $\sigma(i, j)[t]$ to describe whether there is an edge between agent i and agent j at time t and its definition is expressed as follows:

$$\sigma(i,j)[t] = \begin{cases} 0, & if\,((\sigma(i,j)[t^-]=0) \cap (\| q_i - q_j \| \geq r-\varepsilon)) \\ & \cup\,((\sigma(i,j)[t^-]=1) \cap (\| q_i - q_j \| \geq r)) \\ 1, & if\,((\sigma(i,j)[t^-]=1) \cap (\| q_i - q_j \| < r)) \\ & \cup\,((\sigma(i,j)[t^-]=0) \cap (\| q_i - q_j \| < r-\varepsilon)) \end{cases} \quad (4)$$

where $\varepsilon \in (0, r)$ is a constant. Hence, the edges are generated by $e(t) = \{(i, j) : \sigma(i, j) = 1, i, j \in v\}$. Then the neighbor set of the agent i at time t is defined as: $N_i(t) = \{ j \mid (i, j) \in e, j \neq i, j = 1, 2, \ldots, n \}$.

Most existing works generally investigate the flocking under the assumption of the connectivity, initial connectivity or the joint connectivity. When the initial network is connected, some existing methods [3,14] adopted the artificial potential function (5) to guarantee the network connectivity.

$$\psi(\| q_{ij} \|) = \frac{r}{\| q_{ij} \| \, (r - \| q_{ij} \|)}, \; \| q_{ij} \| \in (0, r), \quad (5)$$

However, in the real-world problems, the initial network may usually be disconnected, and therefore, the control methods based on the artificial potential function (5) is limited.

In this paper, a new potential function with a penalty term is constructed in (6) to avoid this weakness of potential function (5)

$$\psi(\| q_{ij} \|) = \sigma \frac{r}{\| q_{ij} \| (r - \| q_{ij} \|)} + (1-\sigma)\,(\| q_{ij} \| - r)^2, \quad (6)$$

where r is the sensing radius, σ is defined in (4). Obviously, by the symmetry of function $\psi(\| q_{ij} \|)$, we have following equation:

$$\nabla_{q_i} \psi(\| q_{ij} \|) = -\nabla_{q_j} \psi(\| q_{ij} \|) = \nabla_{q_{ij}} \psi(\| q_{ij} \|). \quad (7)$$

It is noted that $\psi(\| q_{ij} \|) \to \infty$ when $\| q_{ij} \| \to r^-$ or $\| q_{ij} \| \to 0$. In addition, $\psi(\|q_{ij}\|)$ attains its unique minimum when $\|q_{ij}\|$ is equal to the desired distance. On the one hand, when the node j is out of the sensing radius of node i, we have $\sigma = 0$. Thus $\psi(\| q_{ij} \|) = (\| q_{ij} \| - r)^2$, and $-\partial\psi / \partial q_j > 0$, which indicates that interaction force between nodes j and i is attractive force, and thus node j will move to the node i. On the other hand, if the node j is within the sensing radius of node i, then $\sigma = 1$ and $\psi(\| q_{ij} \|)$ become (5). This indicates that node j will never move out of the sensing radius of node i because $\psi(\| q_{ij} \|) \to \infty$ when $\| q_{ij} \| \to r$.

Remark 1: The first term in the right hand of (6) provides the infinite repulsive force to avoid collision and fragmentation. However, the second term in the right hand of (6) will provide the attractive force to make two agents be closer. The design of potential function can make the whole network be connected even the initial network is disconnected.

In (2), the control input is $u_i = f_i^\alpha + f_i^\beta + f_i^\gamma$, where f_i^α is the local attractive or repulsive force to cope with the separation and cohesion; f_i^β regulates its velocity with its neighbor; f_i^γ is a navigational feedback term to drive the agents to track the virtual leader.

The motion of the virtual leader is described by

$$\dot{q}_\gamma = p_\gamma, \quad \dot{p}_\gamma = f(p_\gamma), \quad (8)$$

where $q_\gamma, f, p_\gamma \in \mathrm{R}^m$ are the position, acceleration, and the velocity of the virtual leader, respectively.

The aim of flocking control with a virtual leader is to design the control input such that

$$0 < \| q_i(t) - q_j(t) \| < \infty, \; \lim_{t\to\infty} \| p_i(t) - p_\gamma(t) \| = 0,$$

for all agent $i = 1, 2, \ldots, n$.

Consider the unknown nonlinear dynamics of the agents, nonlinear measurements or uncertain of the virtual leader, the control input in (2) is specified as

$$u_i(t) = \underbrace{-\sum_{j=1}^{n} \nabla_{q_i} \psi(\| q_{ij} \|)}_{f_i^\alpha} \underbrace{-\rho \sum_{j\in N_i(t)} a_{ij}\, \mathrm{sgn}(p_i - p_j)}_{f_i^\beta} \underbrace{-c_1 h_i (q_i - q_\gamma) - c_2 h_i \phi(p_i - p_\gamma)}_{f_i^\gamma} \quad (9)$$

where $h_i = 1$ if agent i is an informed agent, else $h_i = 0$; function $\mathrm{sgn}(x)$ is the signum function; ρ is a parameter to be determined later, c_1 and c_2 are positive constants. $\phi(\cdot)$ is the uncertain nonlinear feedback of the virtual leader, which is caused by the external disturbances or the errors of the measurements due to the failures or restrictions of speed sensors.

Assumption 2: The nonlinear function $\phi(\cdot)$ satisfies

$$\| \phi(x - y) \|_2 \leq \alpha, \quad \forall x, y \in \mathrm{R}^m \quad (10)$$

where α is a positive constant.

Compared with the existing results [23-25], the assumption 2 indicates that nonlinear velocity measurements of the virtual leader are only needed to be bounded.

In next section, we will present a theoretical analysis to prove the effectiveness of the proposed control protocol. The following results are used for the analysis.

Lemma 1 (Barbalat lemma [29]): Let $f : \mathrm{R}^m \to \mathrm{R}^m$ be a uniformly continuous function on $[0, \infty)$. Suppose that $\lim_{t\to\infty}\int_0^t f(\tau)d\tau$ exists and is finite, then $\lim_{t\to\infty} f(t)=0$.

4. STABILITY ANALYSIS

Define the total energy of the system as follows

$$Q(q,p)=\sum_{i=1}^{n}[U_i(q)+(p_i-p_\gamma)^{\mathrm T}(p_i-p_\gamma)], \quad (11)$$

where $q=[q_1^{\mathrm T},q_2^{\mathrm T},\dots,q_n^{\mathrm T}]^{\mathrm T}\equiv \mathrm{col}(q_1,q_2,\dots,q_n)\in \mathrm{R}^{nm}$, $p=\mathrm{col}(p_1,p_2,\dots,p_n)$ and

$$U_i(q)=\sum_{j=1}^{n}\psi(\|q_{ij}\|)+h_ic_1(q_i-q_\gamma)^{\mathrm T}(q_i-q_\gamma).$$

Clearly, Q is a positive semi-definite function.

Theorem 1: Consider a system of n agents with motion (2) steered by the control protocol (9), assumptions 1 and 2 holds, and the initial energy $Q_0:=Q(t_0)$ is finite. If the parameter ρ in control protocol (9) satisfies $-\rho\sqrt{\lambda_0(t)}+(2c_2\alpha+2\kappa)\sqrt{nm}\le 0$ with $\lambda_0(t)=\min_i\{\lambda_i(L(t))\,|\,\lambda_i>0,\ i=1,2,\dots,n\}$, then we have

(i) All agents will keep the same velocity with the virtual leader asymptotically.

(ii) The system asymptotically approaches to a configuration that is the local minimum of the artificial potentials energy.

(iii) No collision happens between the mobile agents during the evolution.

Proof:

Assume that $G(t)$ switches at time t_l for $l=0,1,2,\dots$. The instants $t_1,t_2,\dots$ are denoted as a series of switching instants such that the topology is invariance within each of the nonempty, bounded, and non-overlapping time-intervals $[t_l,t_{l+1})$ for $l=0,1,\dots$. Furthermore, one has $L(t)=L(t_l),\ t\in[t_l,t_{l+1})$.

(a) We prove part (i).

We first consider the motion of agents in the interval $[t_0,t_1)$. Denote $\tilde q_i=q_i-q_\gamma$, $\tilde p_i=p_i-p_\gamma$ and $\tilde q_{ij}=\tilde q_i-\tilde q_j$, the motion of each agent satisfies

$$\dot{\tilde q}_i=\tilde p_i,\ \ \dot{\tilde p}_i=\tilde u_i, \quad (12)$$

where the control input (9) is rewritten as

$$\begin{aligned}\tilde u_i(t)=&-\sum_{j=1}^{n}\nabla_{\tilde q_i}\psi(\|\tilde q_{ij}\|)-\rho\sum_{j\in N_i(t)}a_{ij}\,\mathrm{sgn}(\tilde p_i-\tilde p_j)\\&-c_1h_i\tilde q_i-c_2h_i\phi(\tilde p_i)+f(p_i)-f(p_\gamma).\end{aligned} \quad (13)$$

Simultaneously, the energy function (11) becomes

$$Q(\tilde q,\tilde p)=\sum_{i=1}^{n}[U_i(\tilde q)+\tilde p_i^{\mathrm T}\tilde p_i], \quad (14)$$

where $\tilde q=[\tilde q_1^{\mathrm T},\tilde q_2^{\mathrm T},\dots,\tilde q_n^{\mathrm T}]^{\mathrm T}$, $\tilde p=[\tilde p_1^{\mathrm T},\tilde p_2^{\mathrm T},\dots,\tilde p_n^{\mathrm T}]^{\mathrm T}$ and

$$U_i(\tilde q)=\sum_{j=1}^{n}\psi(\|\tilde q_{ij}\|)+h_ic_1\tilde q_i^{\mathrm T}\tilde q_i. \quad (15)$$

Because of the symmetry of ψ and $A(G)$, the time derivative of Q along the trajectories of the agents is given by

$$\dot Q(\tilde q,\tilde p)=\sum_{i=1}^{n}2\tilde p_i^{\mathrm T}\tilde u_i+\sum_{i=1}^{n}\sum_{j=1}^{n}\dot\psi(\|\tilde q_{ij}\|)+\sum_{i=1}^{n}2h_ic_1\tilde p_i^{\mathrm T}\tilde q_i. \quad (16)$$

By (7), we have

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}\dot\psi(\|\tilde q_{ij}\|)\\&=\sum_{i=1}^{n}\sum_{j=1}^{n}\dot{\tilde q}_i^{\mathrm T}\nabla_{\tilde q_{ij}}\psi(\|\tilde q_{ij}\|)-\sum_{j=1}^{n}\sum_{i=1}^{n}\dot{\tilde q}_j^{\mathrm T}\nabla_{\tilde q_{ij}}\psi(\|\tilde q_{ij}\|)\\&=\sum_{i=1}^{n}\sum_{j=1}^{n}\dot{\tilde q}_i^{\mathrm T}\nabla_{\tilde q_i}\psi(\|\tilde q_{ij}\|)+\sum_{j=1}^{n}\sum_{i=1}^{n}\dot{\tilde q}_j^{\mathrm T}\nabla_{\tilde q_j}\psi(\|\tilde q_{ij}\|)\\&=2\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde p_i^{\mathrm T}\nabla_{\tilde q_i}\psi(\|\tilde q_{ij}\|).\end{aligned} \quad (17)$$

Substituting (13) and (17) into (16), the following equation can be obtained

$$\begin{aligned}\dot Q(\tilde q,\tilde p)=&-\rho\sum_{i=1}^{n}2\tilde p_i^{\mathrm T}[\sum_{j\in N_i(t)}a_{ij}\,\mathrm{sgn}(\tilde p_i-\tilde p_j)]\\&+\sum_{i=1}^{n}2\tilde p_i^{\mathrm T}[-c_2h_i\phi(\tilde p_i)+f(p_i)-f(p_\gamma)].\end{aligned} \quad (18)$$

Due to the symmetry of $A(G)$, we can get

$$\begin{aligned}&-\rho\sum_{i=1}^{n}2\tilde p_i^{\mathrm T}[\sum_{j\in N_i(t)}a_{ij}\,\mathrm{sgn}(\tilde p_i-\tilde p_j)]\\&=-\rho[\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\tilde p_i^{\mathrm T}\mathrm{sgn}(\tilde p_i-\tilde p_j)+\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\tilde p_i^{\mathrm T}\mathrm{sgn}(\tilde p_i-\tilde p_j)]\\&=-\rho[\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\tilde p_i^{\mathrm T}\mathrm{sgn}(\tilde p_i-\tilde p_j)+\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ji}\tilde p_j^{\mathrm T}\mathrm{sgn}(\tilde p_j-\tilde p_i)]\\&=-\rho[\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\tilde p_i^{\mathrm T}\mathrm{sgn}(\tilde p_i-\tilde p_j)-\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ji}\tilde p_j^{\mathrm T}\mathrm{sgn}(\tilde p_i-\tilde p_j)]\\&=-\rho\sum_{i=1}^{n}\sum_{j\in N_i(t)}a_{ij}(\tilde p_i-\tilde p_j)^{\mathrm T}\mathrm{sgn}(\tilde p_i-\tilde p_j).\end{aligned} \quad (19)$$

Hence, the derivative of energy function can be written as

$$\begin{aligned}\dot Q(\tilde q,\tilde p)=&-\rho\sum_{i=1}^{n}2\tilde p_i^{\mathrm T}[\sum_{j\in N_i(t)}a_{ij}\,\mathrm{sgn}(\tilde p_i-\tilde p_j)]\\&+\sum_{i=1}^{n}2\tilde p_i^{\mathrm T}[-c_2h_i\phi(\tilde p_i)+f(p_i)-f(p_\gamma)]\\=&-\rho\sum_{i=1}^{n}\sum_{j\in N_i(t)}a_{ij}(\tilde p_i-\tilde p_j)^{\mathrm T}\mathrm{sgn}(\tilde p_i-\tilde p_j)\\&-\sum_{i=1}^{n}2c_2h_i\tilde p_i^{\mathrm T}\phi(\tilde p_i)+\sum_{i=1}^{n}2\tilde p_i^{\mathrm T}(f(p_i)-f(p_\gamma)).\end{aligned} \quad (20)$$

It is obvious that the signs of $f(p_i)-f(p_\gamma)$ and $\phi(\tilde{p}_i)$ are uncertain. But we can conclude that

$$\begin{aligned}
&\dot{Q}(\tilde{q},\tilde{p})\\
&\le -\rho\sum_{i=1}^{n}\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i-\tilde{p}_j)^{\mathrm{T}}\operatorname{sgn}(\tilde{p}_i-\tilde{p}_j)\\
&\quad +2c_2\sum_{i=1}^{n}|h_i\tilde{p}_i^{\mathrm{T}}\phi(\tilde{p}_i)|+2\sum_{i=1}^{n}|\tilde{p}_i^{\mathrm{T}}(f(p_i)-f(p_\gamma))|\\
&\le -\rho\sum_{i=1}^{n}\sum_{j\in N_i(t)} a_{ij}(\tilde{p}_i-\tilde{p}_j)^{\mathrm{T}}\operatorname{sgn}(\tilde{p}_i-\tilde{p}_j)\\
&\quad +2c_2\sum_{i=1}^{n}|\tilde{p}_i^{\mathrm{T}}\phi(\tilde{p}_i)|+2\sum_{i=1}^{n}|\tilde{p}_i^{\mathrm{T}}(f(p_i)-f(p_\gamma))|\\
&\le -\rho\left\|D^{\mathrm{T}}\otimes I_m\tilde{p}\right\|_1+2c_2\sum_{i=1}^{n}\|\tilde{p}_i\|_2\|\phi(\tilde{p}_i)\|_2\\
&\quad +2\sum_{i=1}^{n}\|\tilde{p}_i\|_2\|f(p_i)-f(p_\gamma)\|_2.
\end{aligned}$$

It follows from Assumption 1, Assumption 2 and the properties of Laplacian matrix that

$$\begin{aligned}
&\dot{Q}(\tilde{q},\tilde{p})\\
&\le -\rho\sqrt{\tilde{p}^{\mathrm{T}}(DD^{\mathrm{T}}\otimes I_m)\tilde{p}}+2c_2\alpha\sum_{i=1}^{n}\|\tilde{p}_i\|_2+2\kappa\sum_{i=1}^{n}\|\tilde{p}_i\|_2\\
&\le -\rho\sqrt{\lambda_0(L(t_0))\tilde{p}^{\mathrm{T}}\tilde{p}}+(2c_2\alpha+2\kappa)\sum_{i=1}^{n}\|\tilde{p}_i\|_1\\
&= -\rho\sqrt{\lambda_0(L(t_0))}\,\|\tilde{p}\|_2+(2c_2\alpha+2\kappa)\|\tilde{p}\|_1\\
&\le (-\rho\sqrt{\lambda_0(L(t_0))}+(2c_2\alpha+2\kappa)\sqrt{nm})\|\tilde{p}\|_2\ ,
\end{aligned} \tag{21}$$

where $\lambda_0(L(t_0))=\min_i\{\lambda_i(L(t_0))\,|\,\lambda_i>0,\ i=1,2,\dots,n\}$.

If the parameter ρ is chosen such that

$$-\rho\sqrt{\lambda_0(L(t_0))}+(2c_2\alpha+2\kappa)\sqrt{nm}\le 0, \tag{22}$$

we can get $\dot{Q}\le 0$ in the interval $[t_0,t_1)$ from (21), which indicates

$$Q(t)\le Q(t_0)<\infty,\quad \forall t\in[t_0,t_1)\ . \tag{23}$$

From (23) and the definition of potential function, it is clear that no collision happens and the existing edges will not lost in the interval. Hence, new edges may be added to the network at the switching constant t_1. The switching of potential function (6) causes the discontinuities in energy function. For the boundedness of energy function, we only need to consider the gain for the change of energy function. Based on the hysteresis method, when m_k new edges added at switching time t_1, one has $Q(t_1)=Q(t_1^-)+m_k\psi(\|r-\varepsilon\|)$. Since there are at most $(n-1)(n-2)/2$ new edges that can be connected in graph G, the upper bound of $Q(t_1)$ is finite or $Q(t_1)<\infty$. Applying the similar discussions to t_l for $l=2,3,\dots$, it can be concluded that $Q(t_l)$ is bounded for any t_l, $l=2,3,\dots$.

Applying the similar discussions to the interval $t\in[t_l,t_{l+1})$, $l=1,2..$, the derivative of energy function at each $t\in[t_l,t_{l+1})$ is given by

$$\dot{Q}(t)\le(-\rho\sqrt{\lambda_0(L(t_l))}+(2c_2\alpha+2\kappa)\sqrt{nm})\|\tilde{p}\|_2\le 0. \tag{24}$$

Consequently, it follows from the assumption of the parameter ρ and (24) that

$$Q(t)\le Q(t_l)<\infty,\ \text{for } t\in[t_l,t_{l+1}),\ l=2,3,\dots. \tag{25}$$

At each interval $[t_l,t_{l+1})$, $Q(t)$ decreases with an increase of time, which implies that all agents gradually adjust their position to the desired distance and their velocity to that of the virtual leader. After a finite time, the topology is fixed when all edges are connected.

The above analysis shows that $Q(t)$ is bounded all the time. Therefore, for $t\ge 0$, there must be a set

$$\Omega=\left\{\left[\tilde{q}^{\mathrm{T}},\tilde{p}^{\mathrm{T}}\right]^{\mathrm{T}}\in\mathrm{R}^{2nm}\,|\,Q(\tilde{q},\tilde{p})\le\upsilon,\upsilon>0\right\}. \tag{26}$$

It follows from (14) and (26) that $\tilde{p}$ and $\tilde{q}$ are bounded. Define $\Gamma(t)=\|\tilde{p}\|_2$, and it is obvious that $\dot{\Gamma}(t)$ is also bounded. Note that

$$\lim_{t\to\infty}(Q(t)-Q(0))=\int_0^\infty\dot{Q}(t)dt=\sum_{l=0}^{\infty}\int_{t_l}^{t_{l+1}}\dot{Q}(\tau)d\tau\ . \tag{27}$$

Define $\Delta(l)=\int_{t_l}^{t_{l+1}}\dot{Q}(\tau)d\tau$. It is clearly that $\Delta(l)\le 0$ due to $\dot{Q}(t)\le 0$. In (27), the left side of equation is bounded, which indicates that

$$\lim_{l\to\infty}\Delta(l)=0\ . \tag{28}$$

Based on (22) and (24), there exists a positive constant w such that $\dot{Q}(t)\le -w\|\tilde{p}\|_2$, and it will have

$$\Delta(l)=\int_{t_l}^{t_{l+1}}\dot{Q}(\tau)d\tau\le -w\int_{t_l}^{t_{l+1}}\|\tilde{p}(\tau)\|_2\,d\tau\ , \tag{29}$$

and there is further

$$0\le\int_{t_l}^{t_{l+1}}\Gamma(\tau)d\tau\le -\frac{1}{w}\Delta(l)\ , \tag{30}$$

which indicates that

$$\lim_{l\to\infty}\int_{t_l}^{t_{l+1}}\Gamma(\tau)d\tau=0\ . \tag{31}$$

Since

$$\int_0^\infty\Gamma(\tau)d\tau=\sum_{l=0}^{\infty}\int_{t_l}^{t_{l+1}}\Gamma(\tau)d\tau\ , \tag{32}$$

$\lim_{t\to\infty}\int_0^t\Gamma(\tau)d\tau$ exists and is bounded. By the lemma 1, we can get $\lim_{t\to\infty}\Gamma(t)=\lim_{t\to\infty}\|\tilde{p}\|_2=0$, which is

$$p_1=p_2=\cdots=p_n=p_\gamma\ . \tag{33}$$

(b) We now proceed to prove Part (ii).

From the proof of part (i), we see that, in the steady state, $\dot{p}_1=\dot{p}_2=\cdots=\dot{p}_n=f(p_\gamma)$, which implies

$u_i = 0$. It thus follows from (13) and (15) that

$$\nabla_{\tilde{q}}[\sum_{i=1}^{n} U_i(\tilde{q})] = 0,$$

which indicates that the configuration converges asymptotically to a fixed configuration which is an extreme of all agent global potentials.

(c) The proof of part (iii).

Assume any two agents b and s collide at time $t_c \in [t_l, t_{l+1})$, that is $q_b(t_c) = q_s(t_c)$. Then,

$$\begin{aligned} V(q) &= \frac{1}{2}\sum_{j=1}^{n}\psi(\| q_{ij} \|) \\ &= \psi(\| q_b - q_s \|) + \frac{1}{2}\sum_{\substack{j=1 \\ i,j\neq b,s}}^{n}\psi(\| q_{ij} \|) \\ &> \psi(\| q_b - q_s \|). \end{aligned}$$

From the proof of part (a) and the definition of energy function Q, one has

$$\psi(\| q_b - q_s \|) < V(q) < Q < \infty. \quad (34)$$

Note that $\psi(\| q_b - q_s \|) \to \infty$ as $\| q_b - q_s \| \to 0$. Thus (34) contradicts the assumption that $q_b(t_c) = q_s(t_c)$. Therefore, no collision happens during the evolution. The proof of the theorem is completed.

5. NUMERICAL SIMULATIONS

In this section, a simulation is performed on 15 agents moving in a 2-dimensional space. The initial positions and velocities of agents are chosen randomly from $[0,15]\times[0,15]$ and $[0,1]\times[0,1]$, respectively; the sensing radius $r = 4$ with $\varepsilon = 0.1$; $c_1 = 1$ and $c_2 = 1.5$ for the virtual leader. The initial position and velocity of the virtual leader are set as $q_\gamma(0) = [9,9]^{\mathrm{T}}$, $p_\gamma(0) = [0.5, 0.5]^{\mathrm{T}}$. The unknown nonlinear dynamics for the agents and virtual leader are described by $f(p_x, p_y) = [2\sin(w_1 p_x), 2\cos(w_2 p_y)]^{\mathrm{T}}$, where w_1 and w_2 are the unknown numbers and set as $w_1 = w_2 = 1$ in this simulation. The nonlinear function $\phi(x)$ in control (9) is chosen as follows.

$$\phi(x) = \begin{cases} \tanh(x)/\tanh(1) - 0.5 & x > 1 \\ x^2 - 0.5 & 0 < x \le 1 \\ 0.5 - \exp(x) & x \le 0 \end{cases} \quad (35)$$

Clearly, (35) satisfies the assumption 2, but does not satisfy the assumption in [25]. Fig. 1 (a) shows the initial state of all agents which are highly disconnected, and then we choose the agents 1-4 as the informed agents. With an increase of time, the number of connected sub-networks decreases and achieves flocking as shown in Fig. 1 (b). Fig. 2 (a) and Fig. 2 (b) depict the velocity differences on different axis between the agents and the virtual leader, which clearly shows that all agents eventually move with the same velocity as the virtual

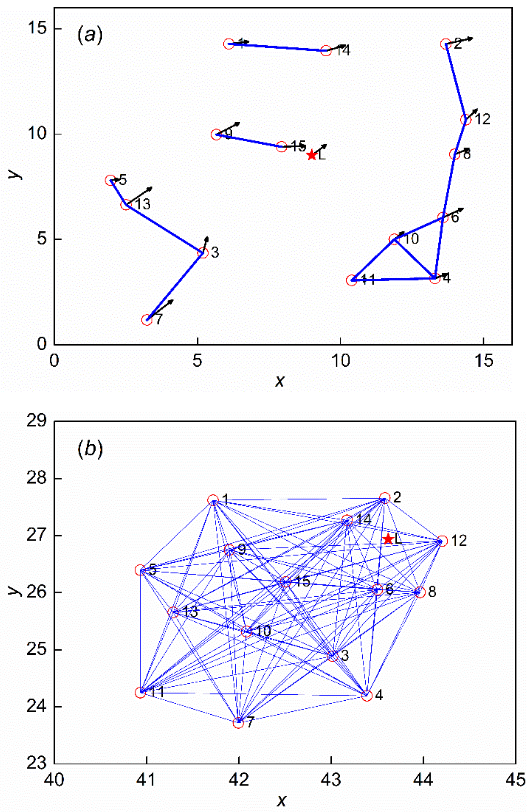

Fig. 1. The initial state and the flocking state of the network.

leader. The trajectories of the agents and the virtual leader are shown in Fig. 3 (a), which indicates that all agents can move ahead with the trajectory of the virtual leader eventually. Fig. 3 (b) records the maximum distance and minimum distance between the agents during the motion. It can be proven from the results in Fig. 3 that there is no collision between any mobile agents.

Furthermore, we make a comparison in Fig. 4 for the convergence between the proposed method, Wen's method [25], Gao's method [16] and Su's method [3], where the initial positions and velocities of the agents are regenerated randomly. For the unconnected initial network, Su's method fails to work for flocking. The corresponding results are shown as the dash-dot lines in Fig. 4, which indicates the divergence. In addition, the velocity measurements $\phi(x)$ do not satisfy the convergent condition $x\phi(x) > 0$ in Wen's method. Wen's method fails to work in the simulation, and the corresponding results have the oscillation as shown in dash lines. Gao's method works and a comparison is made between the proposed method and Gao's method as shown in Fig. 4. Using the proposed method, the velocity difference approaches to zero after about 2000 iterations; while the system achieves flocking after about 3800 iterations when Gao's method is used. It shows that the proposed method has high convergence and broader applicability in the more complex real-world problems.

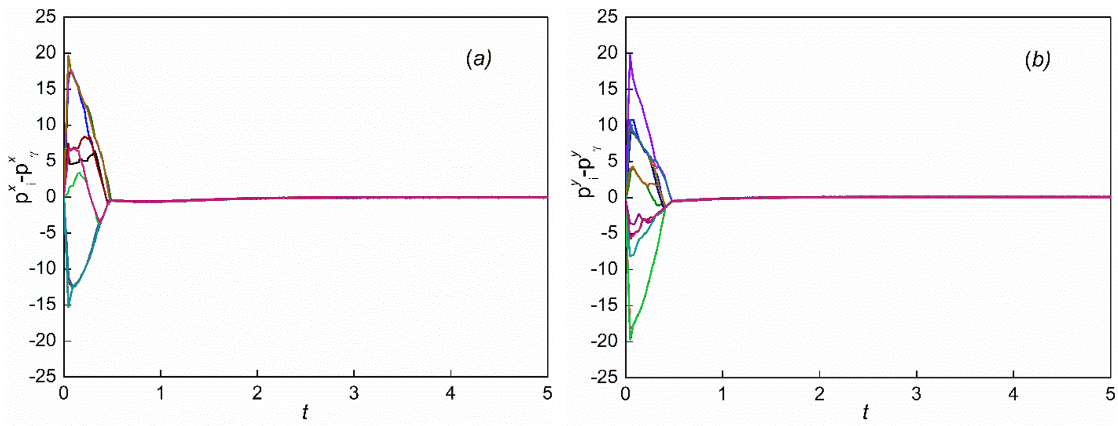

Fig. 2. The velocity differences between the agents and the virtual leader.

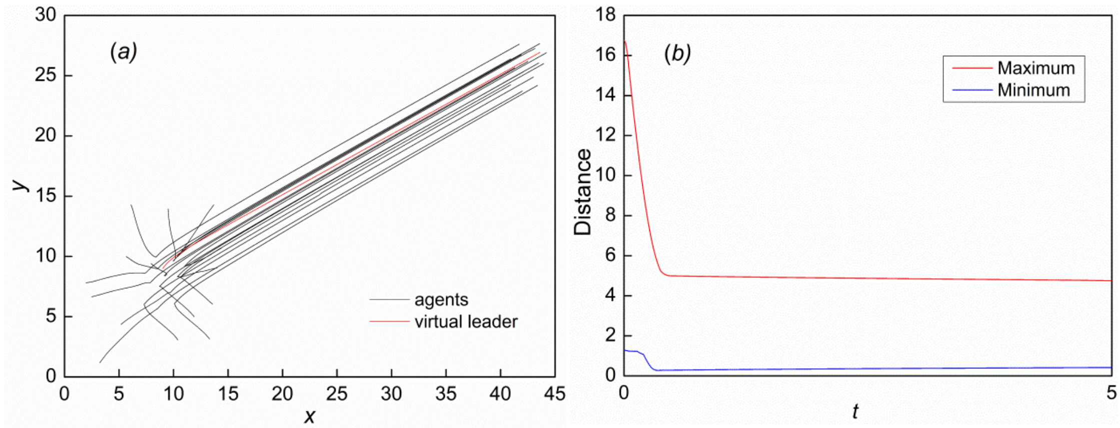

Fig. 3. (a) The motion of the agents and the virtual leader; (b) The maximum distance and the minimum distance between any two agents in the motion.

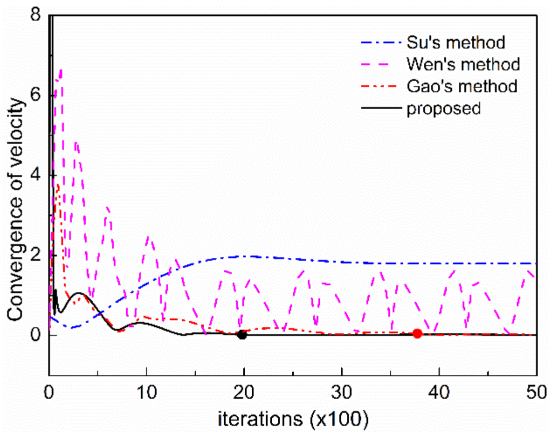

Fig. 4. The comparisons of convergence between the proposed method and the existing methods.

6. CONCLUSIONS

In this paper, the flocking control of nonlinear multi-agent systems with heterogeneous virtual leader is investigated, where the uncertain nonlinearity of the virtual leader information is only bounded. The nonlinear terms of the unknown dynamics satisfy a weaker condition than the Lipschitz condition. A dynamical parameter is employed in control protocol for each agent to achieve the flocking. In addition, a new potential function includes a penalty term is proposed in the design of the control input to guarantee the connectivity when the initial network is disconnected. The Barbalat lemma is employed for the analysis of the flocking, where there is no need to get the state of the minimum total energy. Theoretical analysis demonstrates that all agents can avoid collision and converge to the common velocity. Finally, the effectiveness of the proposed algorithm is clearly verified through simulation experiments. It shows that the proposed method has broader applicability in some practical applications with more stringent restrictions.

REFERENCES

[1] X. Xie, and X. Mu, "Observer-based Intermittent Consensus Control of Nonlinear Singular Multi-agent Systems," *International Journal of Control, Automation, and Systems*, vol. 17, no. 9, pp. 2321-2330, September 2019.

[2] Y. Liu and G. Yang, "Distributed fixed-time consensus for multiagent systems with unknown control directions," *International Journal of Robust*

and Nonlinear Control, vol. 29, no. 11, pp. 3311-3329, July 2019.

[3] H. Su, G. Chen, X. Wang, and Z. Lin, "Adaptive second-order consensus of networked mobile agents with nonlinear dynamics," *Automatica*, vol. 47, no. 2, pp. 368–375, February 2011.

[4] M. Chen, and X. Wang, "Flocking dynamics for multi-agent system with measurement delay," *Mathematics and Computers in Simulation*, vol. 171, pp. 187-200, May 2020.

[5] G. Jing and L. Wang, "Multi-agent Flocking with Angle-based Formation Shape Control," *IEEE Transactions on Automatic Control*, vol. 65, no. 2, pp. 817-823, February 2020.

[6] X. Luo, X. Li, S. Li, Z. Jiang, and X. Guan, "Flocking for Multi-agent Systems with Optimally Rigid Topology Based on Information Weighted Kalman," *International Journal of Control, Automation, and Systems*, vol. 15, no. 1, pp. 138-148, February 2017.

[7] C. W. Reynolds, "Flocks, herds, and schools: a distributed behavioral model," *Computer Graphics*, vol. 21, no. 4, pp. 25-34, July 1987.

[8] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Stable flocking of mobile agents part II: dynamic topology," *42nd IEEE Conference on Decision and Control*, pp. 2016-2021, December 2003.

[9] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401-420, March 2006.

[10] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in Fixed and Switching Networks," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 863-868, May 2007.

[11] J. Zhang, Y. Wang, J. Xiao, and Y. Shen, "Stability analysis of switched positive linear systems with stable and unstable subsystems," *International Journal of Systems Science*, vol. 45, no. 12, pp. 2458-2465, December 2014.

[12] X. Wang, and G. Yang, "Fault-Tolerant Consensus Tracking Control for Linear Multiagent Systems Under Switching Directed Network," *IEEE Transactions on Cybernetics*, vol. 50, no. 5, pp. 1921-1930, May 2020.

[13] M. M. Zavlanos, A. Jadbabaie, and G. J. Pappas, "Flocking while preserving network connectivity," *46th IEEE Conference on Decision and Control*, pp. 2919-1279, December 2007.

[14] H. Su, X. Wang, and G. Chen, "A connectivity-preserving flocking algorithm for multi-agent systems based only on position measurements," *International Journal of Control*, vol. 82, no. 7, pp. 1334-1343, July 2009.

[15] S. Su and Z. Lin, "Connectivity enhancing coordinated tracking control of multi-agent systems with a state-dependent jointly-connected dynamic interaction topology," *Automatica*, vol. 101, pp. 431-438, March 2019.

[16] J. Gao, X. Xu, N. Ding, and Q. E. Li, "Flocking motion of multi-agent system by dynamic pinning control," *IET Control Theory & Applications*, vol. 11, no. 5, pp. 714-722, January 2017.

[17] F. Sun, R. Wang, W. Zhu, and Y. Li, "Flocking in nonlinear multi-agent systems with time-varying delay via event-triggered control," *Applied Mathematics and Computation*, vol. 350, pp. 66-77, June 2019.

[18] Q. Zhang, Z. Gong, Z. Yang, and Z. Chen, "Distributed Convex Optimization for Flocking of Nonlinear Multi-agent Systems," *International Journal of Control, Automation, and Systems*, vol. 17, no. 5, pp. 1177-1183, May 2019.

[19] Z. Peng, D. Wang, H. H. T. Liu, and G. Sun, "Neural adaptive control for leader-follower flocking of networked nonholonomic agents with unknown nonlinear dynamics," *International Journal of Adaptive Control and Signal Processing*, vol. 28, no. 6, pp. 479-495, June 2014.

[20] G. Wen, C. L. P. Chen, Y. Liu, and Z. Liu, "Neural Network-Based Adaptive Leader-Following Consensus Control for a Class of Nonlinear Multiagent State-Delay Systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 8, pp. 2151-2160, August 2016.

[21] B. Ning, Q. Han, Z. Zuo, J. Jin, and J. Zheng, "Collective Behaviors of Mobile Robots Beyond the Nearest Neighbor Rules with Switching Topology," *IEEE Transactions on Cybernetics*, vol. 48, no. 5, pp. 1577-1590, May 2018.

[22] Z. Yang, Q. Zhang, and Z. Chen, "Adaptive distributed convex optimization for multi-agent and its application in flocking behavior," *Journal of the Franklin Institute*, vol 356, no. 2, pp. 1038-1050, January 2019.

[23] Z. Yang, Q. Zhang, and Z. Chen, "Flocking of multi-agents with nonlinear inner-coupling functions," *Nonlinear Dynamics*, vol. 60, no. 3, pp. 255-264, May 2010.

[24] H. Atrianfar and M. Haeri, "Adaptive flocking control of nonlinear multi-agent systems with directed switching topologies and saturation constraints," *Journal of the Franklin Institute*, vol. 350, no. 6, pp. 1545-1561, August 2013.

[25] G. Wen, Z. Duan, Z. Li, and G. Chen, "Flocking of multi-agent dynamical systems with intermittent nonlinear velocity measurements," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 16, pp. 1790-1805, November 2012.

[26] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Stable flocking of mobile agents, Part I: fixed topology," *42nd IEEE Conference on Decision and Control*, pp. 2010-2015, December 2003.

[27] J. Li, X. Chen, F. Hao, and J. Xie, "Event-triggered Bipartite Consensus for Multi-agent Systems with Antagonistic Interactions," *International Journal of Control, Automation, and Systems*, vol. 17, no. 8, pp. 2046-2058, August 2019.

[28] M. Fan, H. Zhang, and M. Wang, "Bipartite flocking for multi-agent systems," *Communications in Nonlinear Science & Numerical Simulation*, vol. 19, no. 9, pp. 3313-3322, September 2014.

[29] H. K. Khalil and J. Grizzle, Nonlinear Systems, vol. 3, Prentice hall Upper Saddle River, NJ, 2002.