Determination and optimization of the mathematic model using the non-linear least square and iteration methods

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***Abstract*—The mathematic model of ring-shaped electrostatic sensor is often represented by its spatial sensitivity, which is defined as the ratio between the induced charge on the electrode to the charge carried by a particle at a different location in the sensing zone. The first step of this study was carried out to investigate the response of the electrode to a charged particle moving axially and radially [1] through simulation, and the second step was to optimize the mathematic model using the non-linear least square and iteration methods. Based on the response to single charge particles, the spatial response of the electrode to a flow stream at different radial position was derived. The purpose of modelling was to establish an accurate analytical expression of the sensor response to single charged particles and flow streams for further study of spatial sensitivity compensation.**

Keywords—Spatial sensitivity, Electrostatic sensor, mathematic model.

I. Introduction

In coal fired power stations and other pneumatically conveyed system, e.g. in blast furnaces, pulverized coal is delivered through a main hopper, and it may be split several times before entering combustors. The flow rate balance among conveyors is not naturally achieved. Accurate flow rate measurement of pulverized coal is essential to the efficiency and safety of combustion, also vital for reducing pollutions [2]. However due to the combination of uneven flow profiles and non-ideal characteristics of sensors, the accurate measurement of uneven dispersed gas-solids two-phase flow has not yet achieved.

Gas-solids flow often takes uneven flow profiles. The sensors for such flow require uniform spatial sensitivity so that the measurement would be independent of flow profile. There are two ways to achieve this goal, i.e. through complex sensor structure or through signal processing. The latter is preferred as it is a low cost approach. This project is dealing with spatial sensitivity of a ring-shaped electrostatic sensor with inherent non-uniform spatial sensitivity, and the aim is to achieve an uniform spatial sensitivity through signal processing [3]. The work presented in this paper is the part of this project. i.e., the mathematic modelling and optimization.

II. Modelling of ring-shaped electrostatic sensor

On a ring-shaped electrostatic electrode, the charge is induced if a charged particle is within the vicinity of the electrode. The induced charge is not only dependent on the charge carried by the particle, but also on the location of it [4].

The induced charge Q due to a particle carrying unit charge can be expressed with the following equation known as the “spatial sensitivity”, which was obtained based on the simulation using Finite Element Method (FEM) [1].

$$Q = Ae^{-kx^2} \tag{1}$$

In the above equation, it is assumed that the unit point charge is at (x, θ,r) in a cylindrical coordinate system, where x denotes the distance along pipeline from the center of the electrode cross section , r is the radius from the pipe central line, θ is the angle to a reference radius. Due to spatial symmetrical characteristics of the ring shaped electrode, θ the angle does not affect the response, so that Q depends on r and x only. Parameters A and k are dependent on electrode geography and r the radial location of the charged particle.

The dependence of the signal upon the particle location has been examined by simulation. The location of the point particle with unity charge was changed along pipeline for a given radial position. The results have been collected for different radial positions with pixel of 2mm in x direction.

III. DERIVATION OF ITERATION EQUATIONS UNDER LEAST-SQUARE CRITERIA

Based on the simulation results, it was confirmed that the curve of induced charge versus axial location x for a given radial position r looks like a Gaussian or “Bell” function as Cheng concluded[1]. However in order to optimize the model, i.e. determine parameter A and k at different radii, the following mathematic derivation has been conducted:

Assume the theoretical or inherent relationship between the induced charge Q by a point charged particle with the unit charge at location x for a given r can be expressed using “(1)”.

For a given radial location r, the simulation result Qi at each location xi was considered as a test result with error. The difference between the simulation results and the results obtained from “(1)” at each x_i was squared. The sum of these squared errors is expressed as δ below,

$$\delta = \sum_{i=1}^{n} (Qi - Ae^{-kxi^2})^2$$

To find optimized factors A and k for a given r under the least square criteria, the partial derivatives of δ to A and to k were performed, hence,

$$\frac{d\delta}{dA} = 2\sum_{i=1}^{n}\left(Qi - Ae^{-kxi^2}\right)\left(-e^{-kxi^2}\right) = 0$$

i.e.

$$\sum_{i=1}^{n} Qi\, e^{-kxi^2} - A\sum_{i=1}^{n} e^{-2kxi^2} = 0$$

It leads to the equations

$$A = \frac{\sum_{i=1}^{n} Qi\, e^{-kxi^2}}{\sum_{i=1}^{n} e^{-2kxi^2}} \qquad (2)$$

The partial derivative to k is given by,

$$\frac{d\delta}{dk} = 2\sum_{i=1}^{n}\left(Qi - Ae^{-kxi^2}\right)\left(-Ae^{-kxi^2}\right)\left(-xi^2\right) = 0$$

$$A\sum_{i=1}^{n} Qi\; xi^2 e^{-kxi^2} - A^2\sum_{i=1}^{n} xi^2 e^{-2kxi^2} = 0$$

$A \neq 0$

$$\sum_{i=1}^{n} Qi\; xi^2 e^{-kxi^2} - A\sum_{i=1}^{n} xi^2 e^{-2kxi^2} = 0$$

The solution of the above equation is,

$$A = \frac{\sum_{i=1}^{n} Qi\; xi^2 e^{-kxi^2}}{\sum_{i=1}^{n} xi^2 e^{-2kxi^2}} \qquad (3)$$

Apparently, there is no direct analytical solution for k, and k is buried in the expression of A in “(2)” and “(3)”. Nonetheless, from two equations, two parameter can be uniquely determined.

A. *Initial values of A and k*

To initialize the iteration based on “(2)” and “(3)”, the initial A or k needs to be obtained. As it is assumed that the relationship between Q and x is governed by “(1)” for a given radius r. It can be seen that an initial parameter A can be found at x=0, and initial k can also be found using “(4)”, which is derived from “(1)” for x≠0.

$$K_{\circ} = ln\ (\frac{Qi}{A_{\circ}})/xi^2 \qquad (4)$$

TABLE I. INITIAL A AND K

r (mm)	$A_{\circ}$	$k_{\circ}$
0	1.33187E-07	2573.498328
2	1.35536E-07	2958.40431
4	1.38843E-07	3073.714083
6	1.44396E-07	2931.405162
8	1.53744E-07	3150.693077
10	1.73347E-07	4171.067951
12	1.95412E-07	4357.925493
14	2.39325E-07	5570.496382
16	3.36691E-07	9489.201716
18	5.18855E-07	12486.85706

The actual initial parameter k at each given radius r was obtained as an average using “(4)”. The initial parameters at each radius r are given in Table I.

B. *Interation*

With initial $k=k_0$, use “(2)” and “(3)” to recalculate A_0 and k_1; A_1 and k_2 and so on until they converge and satisfy the set accuracy. Table II shows the iteration results for A and k at r=0. The accuracy of convergence was set to 0.1%. It can be clearly seen that both A and k were stabilized.

For other radius positions, the method used is the same.

TABLE II. ITERATION RESULTS AT R = 0 MM

	K(Recorded)		***A(Recorded)***
k_1	2573	A_1	1.20E-07
k_2	3098	A_2	1.259E-07
k_3	3315	A_3	1.28E-07
k_4	3396	A_4	1.29E-07
k_5	3425	A_5	1.29E-07
k_6	3435	A_6	1.29E-07
k_7	3438	A_7	1.29E-07
k_8	3439	A_8	1.29E-07
k_9	3440	A_9	1.29E-07
k_{10}	3441	A_{10}	1.29E-07
k11	3441	A_{11}	1.29E-07
k12	3441	A_{12}	1.29E-07

The model in "(1)" was optimized at 10 different radii r, the results are in Table III.

TABLE III. : OPTIMIZED MODEL PARAMETERS AT DIFFERENT RADII.

Radial position r (mm)	***A***	***k***
0	1.29E-07	3441
2	1.302E-07	3502
4	1.32E-07	3658
6	1.38E-07	4017
8	1.47E-07	4654
10	1.62E-07	5718
12	1.85E-07	8034
14	2.25E-07	12202
16	3.05E-07	23369
18	5.08E-07	60973

The results show that how A and k vary with radius r for a given geometry of the sensor.

IV. RESULT VALIDATION

"Fig.1" provides the simulation results for a 40mm diameter ring-shaped electrostatic meter with 2mm electrode and 3mm insulator. The spatial sensitivity is represented by the induced charge (µC) to a unit charge at different radii r when it moves along the pipe axis x. The gap between points is 2mm.

The most rapid change in the sensitivity occurs when the unity charge close to the pipe wall (r=18mm in the graph).

The values of A and k in Table II and III were calculated under the least square criteria based on the results presented in *"Fig.1"*.

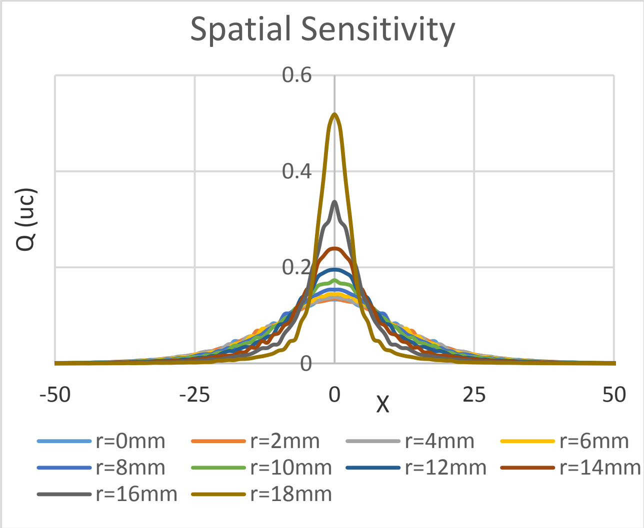

Fig.1: Spatial sensitivity at different radii—simulation results

A. *Validation of the mathematic model*

Fig.2-4 shows the comparisons between the results obtained from the optimized model Q_{eq} and the original simulation data Q(FEM) at r=0, r=6mm, r=18mm.

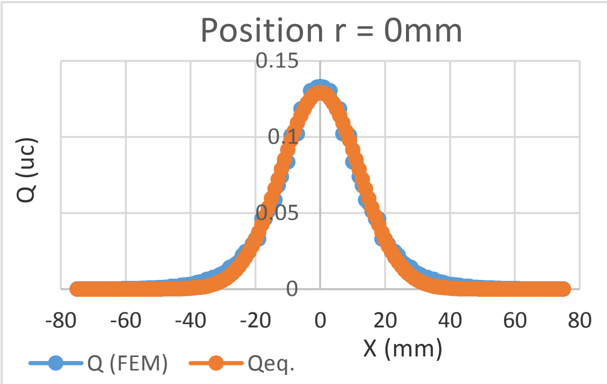

Fig. 2: Model Validation at r = 0mm

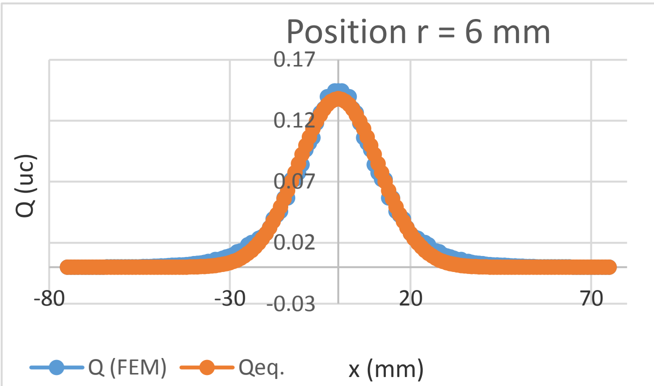

Fig. 3: Model Validation at r=6mm

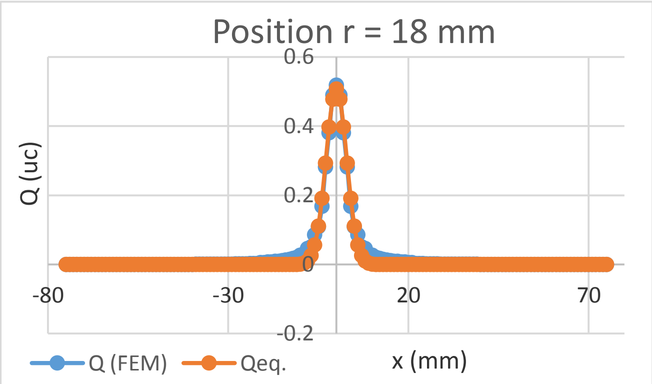

Fig. 4: Model Validation at r = 18mm

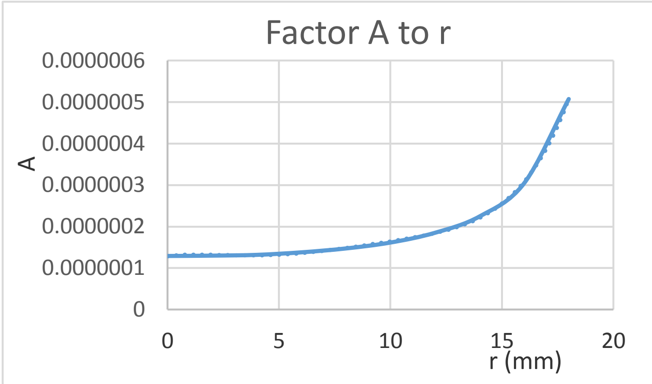

Fig. 5: Variation of A against radial location r

B. Erros of optimised model

With least square method, the results seem well-fit to the optimized model. Table IV provides the Sum of the Square Errors (SSE) at different radii. It can be seen that the model has greater erro when particles passing near pipe wall compared to that when particles near pipe centre. However if SSE is compared to the maximum value, or use the standard deviation to the maximum value ratio, the difference actually is very small.

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TABLE IV. SUM OF SQUARED ERRORS OF FITTING AT DIFFERENT RADII

r	*SSE*	*Standard deviation(σ)*	*σ/Maximum %*
0	7.00193E-16	3.05547E-09	2.29
2	6.35365E-16	2.91059E-09	2.15
4	9.21506E-16	3.50525E-09	2.52
6	9.36063E-16	3.53282E-09	2.45
8	1.16907E-15	3.94811E-09	2.57
10	2.15633E-15	5.362E-09	3.09
12	2.88159E-15	6.19848E-09	3.17
14	4.68948E-15	7.90736E-09	3.30
16	6.42408E-15	9.25497E-09	2.75
18	6.98562E-15	9.65099E-09	1.86

ESTABLISH A UNITY EQUATION

For a given geometry of sensor, it is clear that A and k are function of r. Based on the results on Table III, the relationship between A and r; k and r can be established. "*Fig. 5" and "Fig. 6"* provide graphical results.

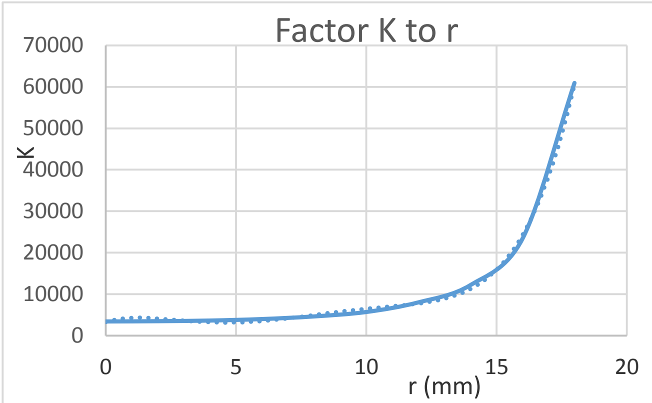

Fig. 6: Variation of k against radial location r

Through the curve-fitting using higher order polynomial function, A can be expressed as

$A=A(r) = 2E\text{-}12r^5\text{-}7E\text{-}11r^4\text{+}9E\text{-}10r^3\text{-}4E\text{-}09r^2\text{+}7E\text{-}09r\text{+}1E\text{-}07$ (4)

with the residual error squared value reaches R^2=0.9997.

From "*Fig.8*", the relationship between k and r can be fitted as,

$K(r)= 0.5056r^5\text{-}18.002r^4\text{+}227.63r^3\text{-}1143.9r^2\text{+}1944.2r\text{+}3260.6$ (5)

The Residual error squared value is R^2=0.9997 as well.

Hence "(1)" can be specified as follows,

$$Q = A(r)e^{-k(r)x^2} \tag{6}$$

Where A(r) and k(r) are expressed in "(4)" and "(5)" for this particular sensor.

RESPONSE TO STREAM

In real application, it is the response to a flow stream in parallel to the sensor central line that is useful. However based on the simulation results to single charged particle depicted in "*Fig.1*", the response to a flow stream can be revealed according to the superposition theorem. The graph in "*Fig.7*" is extracted from "*Fig.1*" for r = 0mm, which represents the induced charge on the electrode when a unit point charge (particle) move from one axial location to another. Hence if we assume a series of particles lie on this radius location evenly, the integration or sum of the response at each different axial location x_i represents the total induced charge due to the stream of particles, assuming each carrying the same charge.

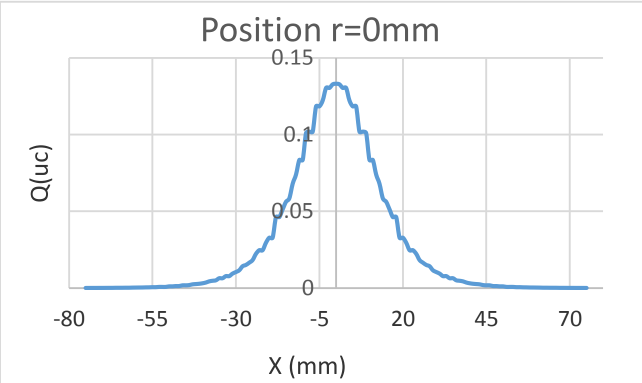

Fig. 7: Variation of induced charge when a unit charge move along pipe axis at r= 0mm.

Use the method explained above, the spatial sensitivity to stream of particles at different radial position has been produced for the sensor studied. This is the sensitivity of the sensor to flow stream, which is show in Fig.8. This is the very crude estimation, more detailed study will be further conducted.

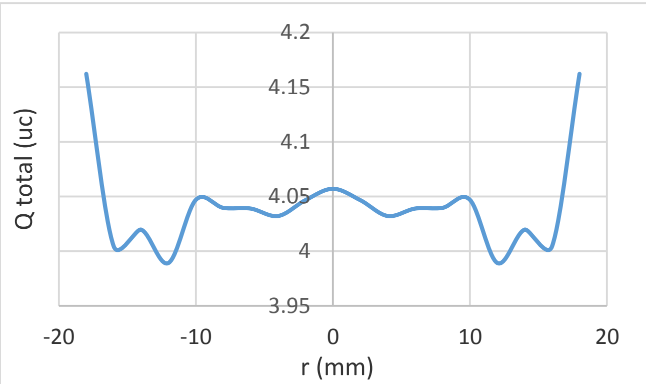

Fig. 8: Spatial sensitivity to flow stream of charged particles

V. DISCUSSION AND CONCLUSIONS

As a part of the project to produce a uniform spatial sensitivity through signal processing, this paper is to deal with the modelling and optimization on a 40mm diameter ring-shaped meter. The Finite Element analysis has been use to produce the original data, and the mathematic model was optimized under the least square criteria. The accuracy of fitting was good. The iteration method was proved to be useful, the parameters of the equation were converged.

A unity equation was derived to represent the induced charge for a particle carrying unit charge at any location of the sensing zone, which is vital for the further research, for example to derive the Fourier transformation of the induced charge, and to study the effect of the radial speed. A expression of the sensitivity to flow stream is proposed, which is practically useful in dealing with inhomogeneous flow profiles, such as "roping".

The model in "(1)" could be optimized by taking logarithm of the equation. However, the results obtained through such process didn't fit for the model. The optimization through logarithm produced an inverted-bell curve, which was not included in the paper. The double exponential model offers more accurate fitting, which is the task currently undertaken, and the results will be reported in a following paper.

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