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The Development and Validation of Multi-axial Creep Damage Constitutive Equations for P91

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Abstract—**The development and validation of multi-axial creep damage constitutive equations for A369 FP91 steel at 625°C was presented. Three aspects of research work were reported: 1) the development of a new set of creep damage constitutive equations based on Xu's formulation [1], 2) the validation on this newly developed and an existing KRH type of constitutive equations were validated under plane stress and plane strain conditions, and 3) discussion and conclusion.**

Keywords: creep damage; constitutive equations; validation; multi-axial formulation

I. INTRODUCTION

Creep design of metallic components operating at high temperature is largely carried out using design code/methods (e.g. BS 5500, and ASME Subsection NH). Flexibility is provided with some codes (e.g. BS 5500) for the designer to use by analysis methods and it has been explored with creep continuum damage mechanics together with the finite element method.

The phenomenological approach of creep damage mechanics can be broadly classified into weak coupling and strong coupling between damage and deformation. In the case of weak coupling the effect of material damage in elastic properties is disregarded and a coupling is established by introducing the damage variables into the constitutive equations with the concept of effective states variables. Within the weak coupling approach, a set of creep damage constitutive equations for uni-axial tension is generalised for multi-axial applications. The success of this approach depends on the development of a set of appropriate constitutive equations capable of depicting the observed multi-axial material behaviour, which will be assessed through validation. That is the concern of this paper.

Specifically to the formulation and validation methodology, Xu proposed a new formulation for multi-axial creep damage constitutive equations [1, 2] and also proposed an improved validation methodology [2] and reported its application to 0.5Cr0.5Mo0.25V ferritic steel [2].

It was noted very limited work published on the review of constitutive equations except a recent publication [3]. It is evident that there is a degree of confusion between calibration and validation among research community.

The P91 as one of 9Cr martensitic steels was developed as a result of the demand for ferritic steels with higher creep strength, but to-date there has been a somewhat limited in-plant experience of their long-term performance, especially with thick section components such as headers and steam pipework [4].

According to [5] it was reported that reduction of creep rupture strength on high Cr steels was investigated: 1) creep damage in the weld joint preferentially accumulated in fine grain region in heat affected zone (HAZ) where creep deformation resistance is lower than other portions [6]; 2) creep void initiation and growth are accelerated due to multi-axial stress states in the HAZ [7], 3) it was also found [8] that creep rupture life of weld joints failed at the HAZ is approximately 1/5 of the base metal and creep strain concentrated in the HAZ of the weld joint based on a finite element creep analysis using the three materials weld joint specimen model consisting of a base metal, a weld metal and a HAZ. Therefore so-called Type IV cracking occurs in actual components.

Recently, research attempted to use computational creep damage mechanics to investigate problems related to P91 pipes and weldments where different type of creep damage constitutive equations were used [9, 10], however, there was not adequate consideration of validation and probably a degree of confusion about calibration and validation.

This paper reports a review of creep deformation and damage, formulation, validation methodology and practice, the recent progress on the development of new set of creep damage constitutive equations, validation for the specific material.

This paper not only demonstrates the capability of this new set of constitutive equations, but also reveals the deficiency in the KRH type of constitutive equations; this discovery is similar to the finding of previous work on 0.5Cr0.5Mo0.25V ferritic steel [2]. Thus, the author addresses again that in the developing damage mechanics field, it is important and necessary to distil the information and conclusions cautiously prior to developing, accepting, and applying any theory and constitutive equation, either new or old (including this one).

II. CREEP DEFORMATION AND DAMAGE [1]

The creep deformation is typically divided into primary, second, and tertiary creep; while the damage process is often understood (with great simplification) to be a process of nucleation, cavity growth and coalescence. In martensite/ferritic steels damage is often associated with substructure coarsening. Uni-axial constitutive equations capable of describing primary, second and tertiary creep have been developed. If only one damage variable is chosen, the creep strain rate and damage rate by

and (1)

By appropriate selection of the function f and g, as well as the critical value of damage, it is possible to represent the tertiary creep and to produce a stress-lifetime relationship consistent with experimental observations. One of the examples is

$$\dot{\varepsilon} = G\left[\frac{\sigma}{1-\omega}\right]^n \quad \text{and} \quad \dot{\omega} = C\frac{\sigma^{\chi}}{(1-\omega)^{\Phi}} \tag{2}$$

where G, C, n, χ and φ are material constants. The effective stress concept is used and the material is deemed to fail when the value of the damage variable reaches its critical value of 1.

Then primary creep could be included by extra hardening function H, such as being proposed and used. For example, the constitutive equations form for uni-axial condition is given as:

(1)

(4)

(5)

(6)

where A, B, h, H^*, and D are material constants.

A. KRH multi-axial Formulation

The KRH type multi-axial creep damage constitutive equations are given as [10]:

(7)

(8)

(9)

(10)

where υ is the stress state index.

B. New Multi-Axial Formulation

The approach originally proposed by Xu [1, 2] was adopted here and the multi-axial creep damage constitutive equations are given as:

(11)

(12)

(13)

(14)

(15)

C. Specific Forms

(16)

(17)

III. VALIDATION METHOD [1]

First, an adequate validation should address (1) what needs to be assessed and (2) under what conditions. With clear understanding of the two fundamental consistency requirements addressed above, it is clear that an adequate validation should be designed and conducted considering:

(1) The items: (a) creep strain rate; and (b) damage evolution;

(2)The stress states: (a) creep curves under uni-axial conditions; (b) multi-axial stress states under proportional loading conditions; and (c) multi-axial states of stress under non-proportional loading conditions.

If a set of creep damage equations is integrated from virgin state to failure, it will produce:

$$\omega = \omega_f, \qquad t = t_f^{multi}, \qquad \bar{\varepsilon} = \bar{\varepsilon}_f^{multi} \tag{18}$$

where ω_f is the critical value of damage and is effective creep strain. These results will be used in validation.

Ideally, the conditions should include proportional and non-proportional loading under multi-axial stress states. Compromise may have to be made due to the constraint imposed by the difficulty to conduct the required experiments and the cost involved, which is not the same as ignorance. Previous practice was not adequate in either the items to be assessed or the range of states of stress.

Practical validation method is proposed as:

(1) To check isochronous rupture loci under plane stress and plane strain states with proportional loading conditions;

(2) To check strain at failure under plane stress and plane strain states with proportional loading conditions;

(3) To check typical creep curves under plane stress and plane strain states with proportional loading conditions;

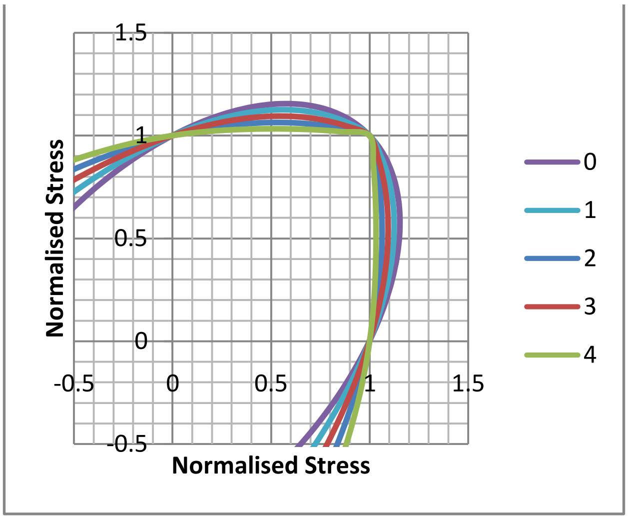
(4) To check the damage development, creep strain development, strain at failure and lifetime for multi-axial stress states complex (or non-proportional) loading condition. One way to achieve this is notched bar test.

In steps 1 and 2, the plane stress and plane strain stress states are selected to present multi-axial stress states under proportional loading conditions. It is suggested that all the material constants should be determined in the first two steps. Step 3 intends to further check the coupling of damage and creep strain. Step 4 validates the constitutive equations under multi-axial non-proportional loading conditions. It is clear that previous practice is not adequate as it ignored the need to include strain at failure under plane stress states with proportional loading conditions and did not consider plane stress states with proportional conditions. This paper will present validation results on the first three accounts.

IV. Result

These two sets of multi-axial creep damage constitutive equations are validated in terms of lifetime, ratios of strain at failure, and creep curves, which corresponds to the first three steps of practical validation method described in above section.

The isochronous rupture loci and ratios of strain at failure for KRH formulation are presented in Figure 1 and Figure 2, respectively, while typical results for the new formulation are presented in Figure 3 and Figure 4. The legends in Figure 1 and Figure 2 are the stress state sensitivity index *v*, while the legends in Figure 3 and Figure 4 are parameter q. Typical results of creep curves and damage evolution under plane stress condition (proportional loading) are shown in Figure 5 and Figure 6. A comparison of creep curves under pure shear condition is given in Figure 7.



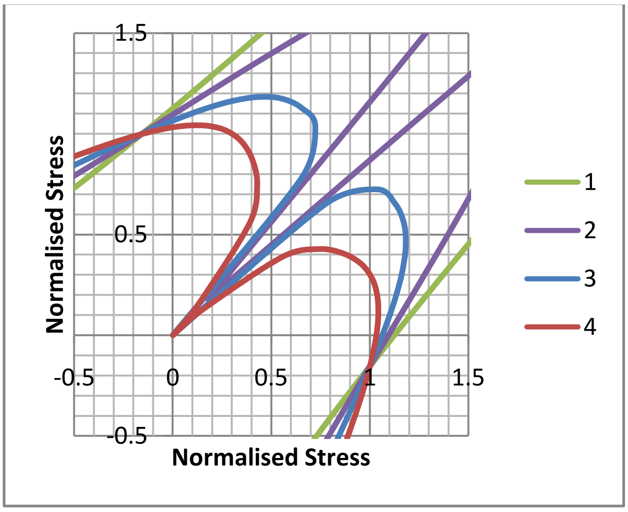


Figure 1. Isochronous rupture loci for KRH formulation (top for plane stress condition and bottom for plane strain condition for *v = 0, 1, 2, 3, and 4*). The legends are for stress state index *v*.

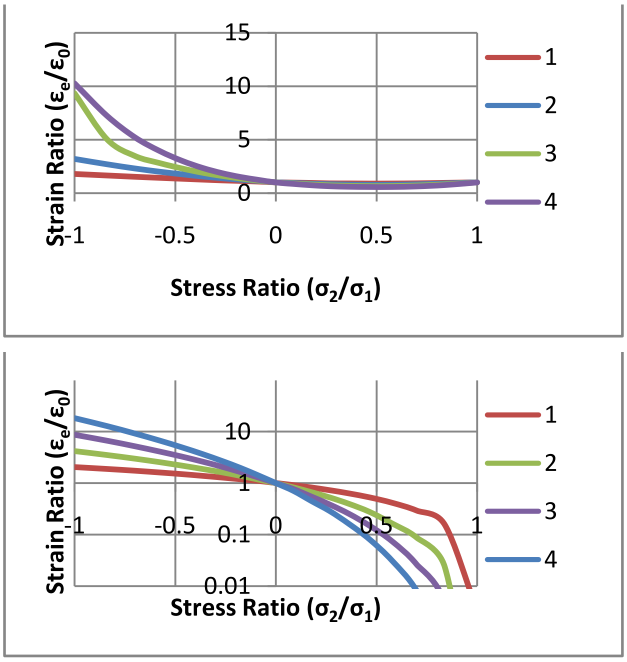


Figure 2. Ratios of strain at failure for KRH formulation (top for plane stress condition and bottom for plane strain condition for *v = 0, 1, 2, 3, and 4*). The legends are for stress state index *v*.

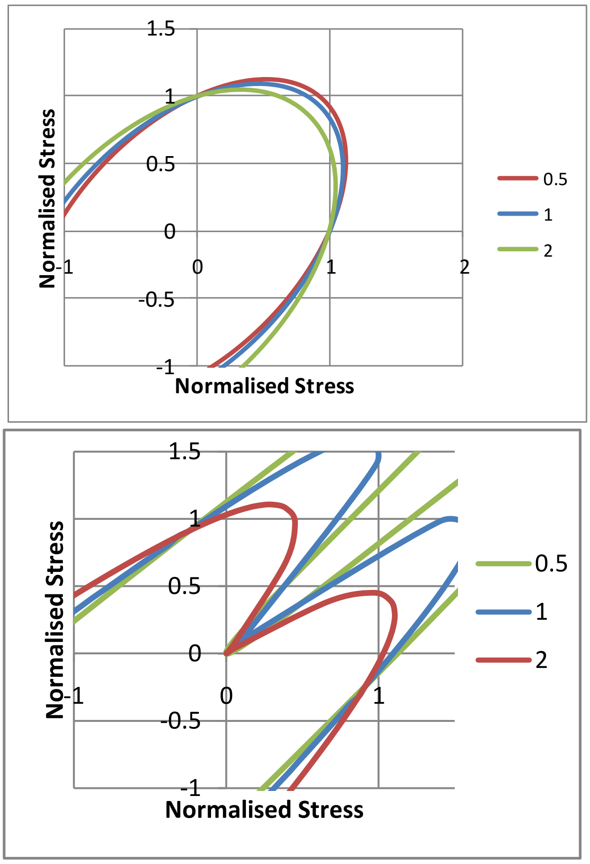


Figure 3. Ratio of strain at failure for new formulation: (a) for plane stress condition and (b) for plane strain condition. A = 0, b = 0, p = 0, q = 0.5, 1 and 2. The legends are for stress parameter q.

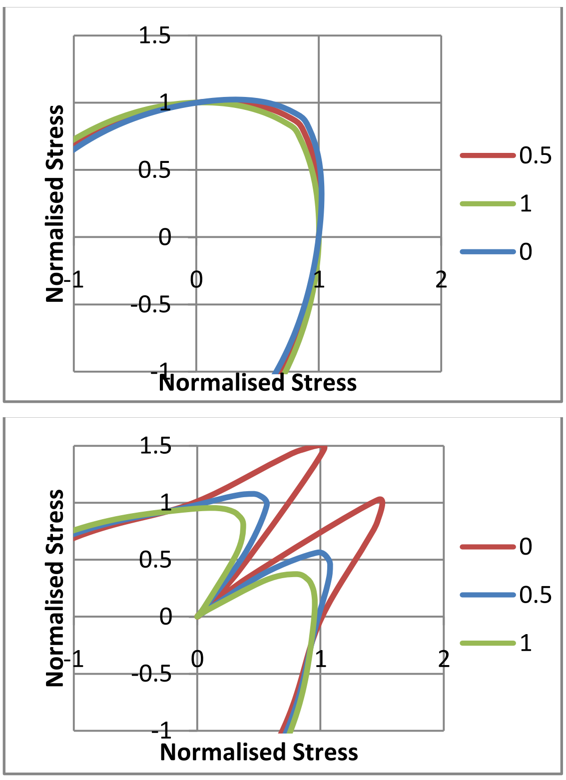


Figure 4. Isochronous rupture loci for new formulation with a = 0, b = 2, p = 1, q = 0, 0.5 and 1. The legends are for stress parameter q.

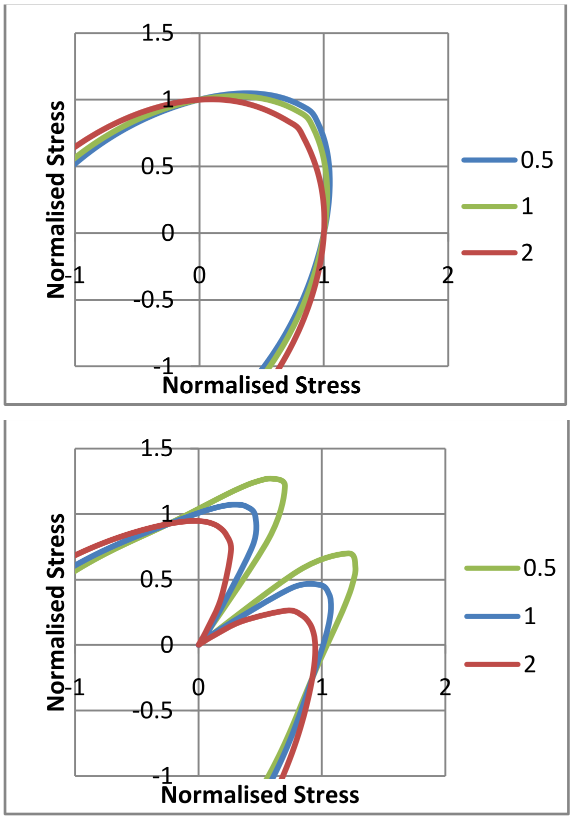
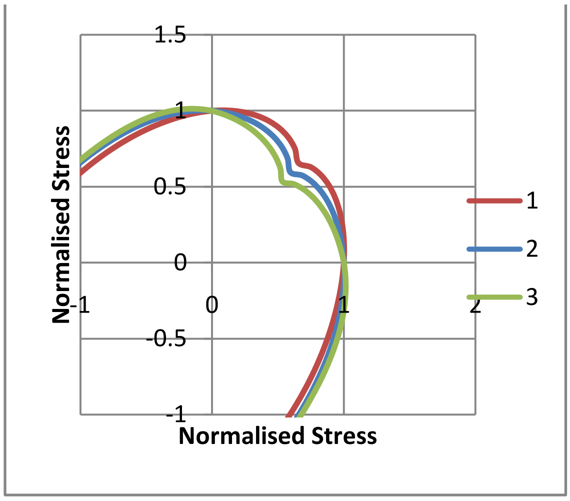


Figure 5. Isochronous rupture loci for new formulation with a = 0, b = 1, p = 1, q = 0.5, 1, and 2. The legends are for stress parameter q.



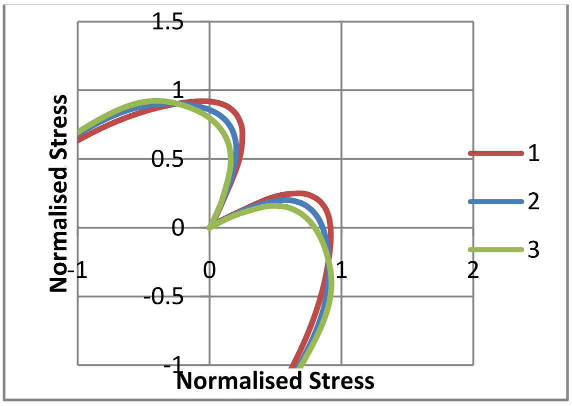


Figure 6. Isochronous rupture loci for new formulation with p = 2.5, q = 1, a = 2 and b =1, 2 and 3. The legends are for stress parameter b.

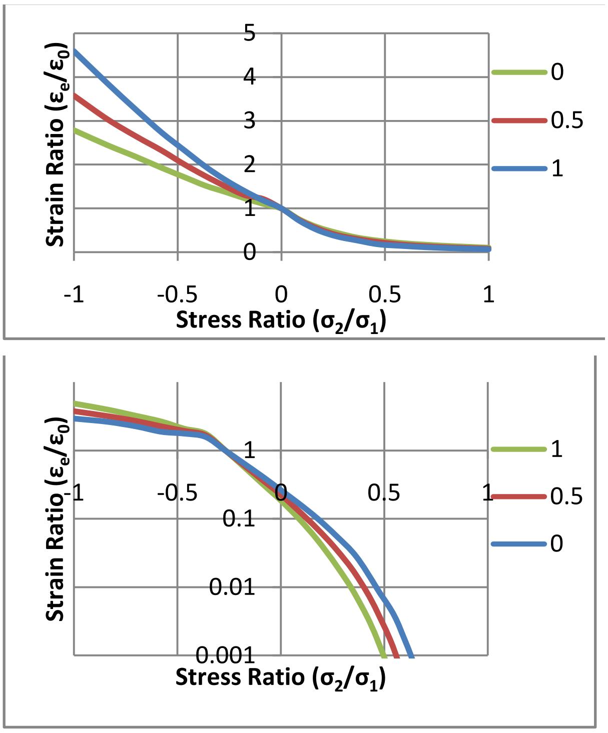


Figure 7. Ratios of strain at failure for new formulation (top for plane stress condition and bottom for plane strain condition with q = 0, 0.5 and 1). The legends are for stress state parameter q.

V. Discussion and Conclusion

A. KRH Formulation

Criterion 1

The plane stress and plain strain isochronous rupture loci (Figure 1) created by using the KRH [11] type of constitutive model and utilised by Hyde et al. [10] demonstrates the inconsistency of the model. Hyde et al. advocate the use of an arbitrary value of one for the stress sensitivity index ν as the claim is made the result is "weakly dependent" upon this value. When the value of ν is one, the plane strain isochronous rupture loci (Figure 2) shows such an increase in creep strength, that this case is simply unrealistic. The plane stress and plane strain isochronous rupture loci demonstrate that the KRH constitutive model cannot simultaneously satisfy the plane stress and plane strain states with a single variable, this is in line with the early finding on a different material [1]. The plane strain state was not included in [11] and no validation at all is reported [10].

The explicit nature of stress state index ν is described in [11] as "a value that minimises the difference between experimental and computed lifetimes". The calibration of the stress sensitivity index ν was accomplished by creating a ratio of actual and computed failure times of several notched bar tests and internally pressurised cylinders. The stress state index (within 10%) of where most of these failures occurred was then proposed as the appropriate value. The analysis in this paper demonstrates this number to be more convenient than representative of the multi-axial stress state.

Criterion 2

The plane stress and plane strain results were plotted as a ratio of principal stresses and strain at failure (Figure 2 and Figure 3). The results further demonstrate the variance and inconsistency as a result of using the stress state index. The range of the plots is more representative of the stress sensitivity index and its mathematical effect, rather than the relative strain at these points. As the value ν increases, the variance in terms of strain becomes increasingly evident and unrealistic.

Criterion 3

The KRH formulation will produce equal life time for uni-axial tension and bi-axial tension which is general inconsistent with experimental observation.

B. Xu's Formulation

Evaluating the results determined by the Xu formulation using the validation methodology:

Criterion 1

A range of plane stress and plane strain isochronous rupture loci are shown in Figs 4 to 5. The optimal results were determined in this analysis using the values proposed by Xu [1] is shown in Figure 6. The plane stress and plane strain isochronous rupture loci (Figure 8 and Figure 9) simultaneously satisfy the plane stress and plane strain states with consistent values of normalised stress in both cases.

Criterion 2

Figure 7 plots the range of stress ratios determined by the Xu's formulation. The plots demonstrate a good correlation between the strain at failure across the range of stress ratios and higher accuracy over the range of the analysis.

Criterion 3

The biaxial tension lifetime is significantly short than that of uni-axial one.

C. Summary

Krausz and Krausz summarise their findings in a review of constitutive models and state "there are many different facets of the same problem and as many answers, the right one is the one that gives the most practical solution, the one that best serves the specific situation" [12]. When compared, the results of the analyses demonstrate a significant increase in the accuracy of the constitutive model and the data produced fully supports this. The isochronous rupture loci reveal the deficiency in RKH formulation and demonstrate the improvement obtained in new formulation proposed by Xu. The criteria of the validation methodology ensured a consistent and fair comparison between the models with demonstrable results.

It is strongly emphasized the need of Experimental data and analysis to satisfy non-proportional loading conditions before the general applicability of a set of constitutive equations is applied for general case study.

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