LMI-Based Design of Robust Gain-Scheduling Control for Precise Slow Motion in a Servodrive

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***Abstract*- this paper presents a new control design method to solve the problem of uncertain friction compensation in robot joint control. A friction force observer is designed using local models of friction interpolated by mean of weighting functions. The estimated friction is used in a minor loop for a compensation purpose and the resulting dynamics after compensation are considered for the synthesis of robust gain scheduling control scheme with uncertain friction compensation assumptions. The performance of this method is verified by simulations and experiments performed on a robot joint.**

I. Introduction

Nonlinearities in general and friction specially are seen as a serious issue in mechanical systems, this motivated a lot of research to deal with the problem and propose various compensating strategies of nonlinear friction [1]

So far, many control strategies have been proposed to deal with the friction problem such as observer-based control [2], adaptive friction compensation [3], sliding mode control [4], neural and fuzzy control [5][6]. Some authors have addressed the problem of inexact friction compensation and its effects on the controlled servosystem [7]. Basically, these underlying approaches can be regarded as either model-based techniques requiring a modeling-identification effort [8], or non-model based methods where friction is considered as an unknown disturbance, even if a prior knowledge of friction is required since this latter is a part of the system dynamics [9].

This paper presents a new model structure for the estimation and robust compensation of friction dynamics. An identification procedure is developed and a highly accurate tracking controller is then designed under uncertain compensation assumptions based on LMI (Linear Matrix Inequality) optimization techniques. Simulations studies and experimental results conducted on a robot joint are presented to demonstrate the effectiveness of the proposed modeling and control techniques.

II. System description: Local modeling for slow motion regime

A. Local Modeling Approach of Friction

A general form of a dynamic friction force depending on position x, velocity v and having only one hidden state z can be expressed as follow

$$F = f(z, x, v) \tag{1}$$

The differential equation driving the internal state dynamics is given by

$$\dot{z} = g(z, x, v) \tag{2}$$

Where g and f are nonlinear functions, that may also include hybrid dynamics to describe the complex nature of friction [10].

A mechanical system with friction is governed by the following equation of motion

$$m\frac{dv}{dt} = u - F \tag{3}$$

Where m is the inertia, v the velocity, u the control signal and F represents the friction forces of the system. This model will be used subsequently to describe the robot joint dynamics by neglecting other nonlinear dynamics such as: Coriolis, centrifugal and gravity forces.

Under friction compensation, the following control input is proposed,

$$u = Ku^{*} + \hat{F} \tag{4}$$

Where K is a positive control gain, u^{*} is an optimal control action to be designed under inexact compensation assumption and $\hat{F}$ is meant to be the friction compensation term based on local modeling approach.

The friction forces given by the nonlinear complex function F in (1) will be described by local linear dynamics according to the velocity related operating conditions of the system governed by (3)

$$F = c_i z + d_i v \quad \text{for} \quad v \in \Omega_i \tag{5}$$

Basically, F can be seen as the sum of a stiff and damping force where c_i can be treated as a local coefficient of stiffness since z has the dimension of a displacement and d_i as the local damping factor respectively. ($\Omega_i \subset \Omega$) is defined as a bounded convex set of operating velocities which must depend on the variation of friction dynamics in order to fit the behavior of the real system. It should be emphasized that, generally, the size of the set depends on how fast are the dynamics of the nonlinearity. It is well known from the steady-state characteristics that real friction is highly nonlinear at very low velocities, where the number of sets i should be higher. In the same way and using approximation techniques, (3) can be rewritten as,

$$\dot{z} = -a_i z + b_i v \quad \text{for} \quad v \in \Omega_i \quad \text{with } i = 0,1,\ldots,n \tag{6}$$

Where a_i and b_i are positive quantities defined according to the local behavior of the system with friction. Along the operating range set Ω_i , (5) is characterised by a static amplification term b_i/a_i and the time constant $1/a_i$. At very low velocities defined by Ω_0 , where the system is under micro-sliding motion and the dynamics of the friction force (5) and (6) can be reduced to,

$$\begin{aligned} F &= c_0 z \\ \dot{z} &= v \end{aligned} \quad \text{for} \quad v \in \Omega_0 \tag{7}$$

The comparison of (5) and (6) with (7) yields the parameter $b_i = 1$ and $c_i = c_0$ can be defined inside Ω_0 .

At higher velocities regime, and assuming that the system is operating at almost constant velocities resulting from slow varying inputs, the steady state condition can be satisfied ($\dot{z} \approx 0$), the friction force dynamics are then given by solving the set of equations (5) and (6) for a constant input velocity inside the switching area of two successive domains Ω_i, Ω_{i+1} . Around zero velocity, the level of friction is decided by its steady state value,

$$F_i = \left(b_i \frac{c_i}{a_i} + d_i \right) v \qquad \text{for} \quad v \in \{\Omega_{1^-}, \Omega_{1^+}\} \tag{8}$$

Where $\{\Omega_{1^-}, \Omega_{1^+}\}$ represent the first domain around zero velocity for negative and positive directions respectively. From (8), an identification method is developed to define the parameter a_i for higher velocities using the dynamics of every two adjacent domains.

B. *Identification Based on local Modeling for Slow Motions*

The proposed model given by (5) and (6) describes friction dynamics inside a certain set Ω_i . For simplicity, b_i=1, $c_i = c_0$ and $d_i = d_0$ are kept constant without loss of the capability of the model to describe the main friction features in the whole Ω domain. According to (8), a_i has clearly an influence on both the level and the speed of the friction; a_i will be defined to fit the local level of friction for the assumed steady state conditions.

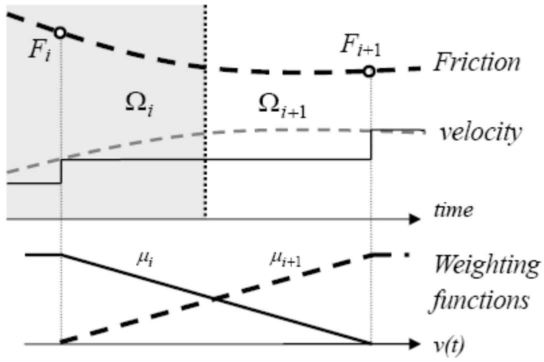


Fig. 1. Identification procedure chart based on local modeling approach.

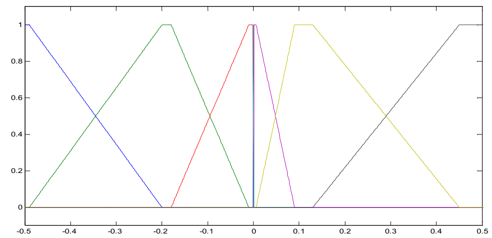


Fig.2. Weighting functions for dynamic switching of local models.

So at t = 0, the pair ($v = v_i$, $F = F_i$) is known from the experimental steady state curve of friction and (8) is solved for $F = F_{i+1}$to yield a_i. For the next adjacent set, $F_i = F_i$+1 is solved for which a_i+1 can be calculated at $t = t_f$, at that time the final value of friction force in the set Ω_i equals to the initial value in the set Ω_{i+1} ,

$$\left(\frac{c_0}{a_{i+1}} \left(1 - e^{-a_{i+1}(t_f)}\right) + d_0 \right) v_{i+1} = \left(\frac{c_0}{a_i} \left(1 - e^{-a_i(t_f)}\right) + d_0 \right) v_i \tag{9}$$

Note that a_i is bounded according to operating velocities of the system, $-\frac{F_{\min}}{c_0} \geq \frac{|v_i|}{a_i} \geq -\frac{F_{\max}}{c_0}$

The smooth transition from a domain to another is ensured by using switching functions that interpolates the model dynamics along the overall operating range Ω ; this will be achieved using weighting functions μ_i as illustrated by Fig.2.

III. FRICTION COMPENSATOR ROBUST DESIGN

The proposed friction state estimator uses the dynamics (5) and takes the form:

$$\dot{\hat{z}} = -a_i \hat{z} + v + l_i u^* \qquad \text{for} \quad v \in \Omega_i \tag{9}$$

Where, l_i is a local compensating gain and $u^* = u/K$ is defined as the control input of the compensated system and will include the tracking error information in the overall controlled system of Fig.3.

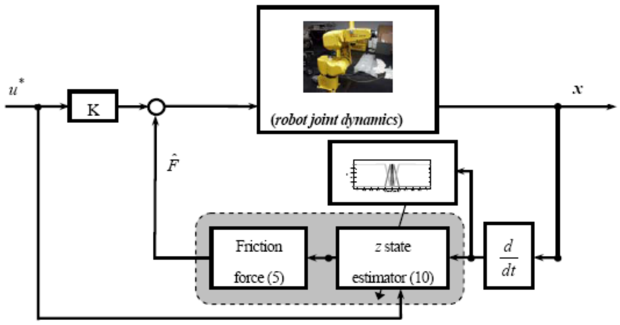


Fig.3. Friction compensation based on Local Modeling Approach in a minor control loop

The observer gains are chosen using local dynamics and (5) to calculate the friction force [11].

The feedback control u^* in (4) can be synthesized using LMI techniques under some severe inexact friction compensation assumptions [12]. By applying the control (4) to the system given by (3), (5) and (6), the dynamics of the pre-compensated system can be formulated as follow:

$$\begin{aligned} \dot{\hat{z}} &= -a_i \hat{z} + v + w_z + l_i u^* \\ \dot{x} &= v \qquad\qquad \text{for} \quad v \in \Omega_i \\ m\dot{v} &= Ku^* - \Delta c_i \hat{z} - \Delta d_i v + w_v \end{aligned} \tag{10}$$

So, the control problem can be stated as follow: Design a stabilizing control law that guarantees performance for the system and taking into account all varying parameters of friction, observer gains and uncertainties resulting from modeling and inexact compensation. This can be formulated through an LMI based optimization procedure and stated as follows: Find,

$$u^* = k_z.z + k_x.x + k_v.v \tag{11}$$

- That minimize $\|T\|_2$ which is the closed loop H_2 norm of the transfer function T from w to $\xi = \alpha x + \beta u^*$, where α , β are weighting coefficient of position and input signal respectively, and their choice is known to be related to performances criteria as well as to the control signal that achieves such performances.
- All closed loop poles lie inside the stable region with a maximum damping value of 0.1.
- subject to the dynamics given by (10) inside all Ω .

Where w_z and w_v include external disturbances and the estimation error which is locally bounded from the design of the observer gains in (9). Regarding the nature of the tracking problem, the system will run into a severe regime including reversal of velocities where static friction has a major influence and stick-slip motions may occur for relatively higher velocities.

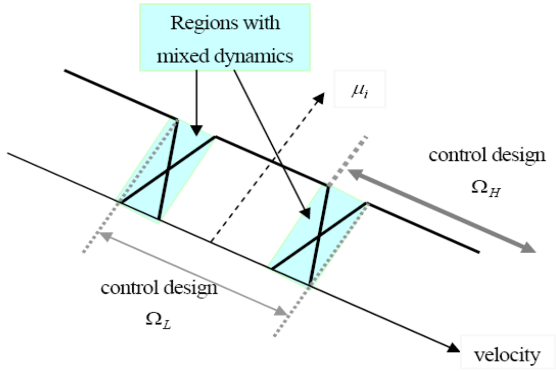


Fig.4. Gain scheduling design based on local modeling approach.

The optimal control problem (11) is then solved for two different overlapping regions Ω_L and Ω_H to generate two state feedback gains, where Ω_L is a symmetric set including zero velocity and Ω_H is the higher velocity set satisfying the following two conditions: (i) $\Omega_L \cup \Omega_H = \Omega$ being the overall set and (ii) $\Omega_L \cap \Omega_H = \Omega_m$ being the region of mixed dynamics corresponding to lower and higher velocities as illustrated by Fig.4.

IV. Simulations and Experimental Evaluation

In this section, simulation results and experiments are conducted to verify the effectiveness of the proposed method. A sinusoidal reference with f=0.1Hz is chosen to ensure tracking slow motions and reversal velocities, regions where friction has a considerable influence. Fig 5 demonstrates a net improvement of the proposed method over a conventional PD control (Fig.6).

The robustness test is performed in simulations by adding a filtered white noise signal to friction that is assumed to detune the friction force level from its nominal value as illustrated by Fig.7.

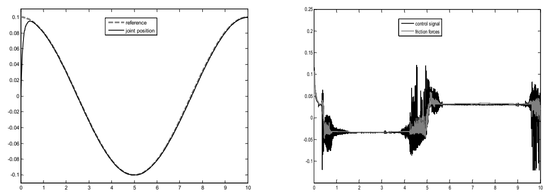


Fig.5. Proposed control without disturbance, Joint position (left), control signal and friction (right).

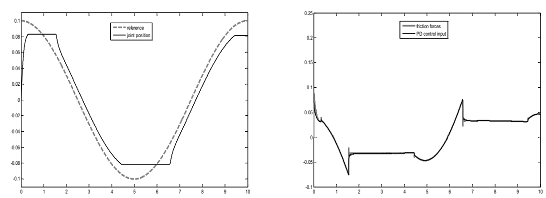


Fig.6. PD control, Joint position (left), control signal and friction (right).

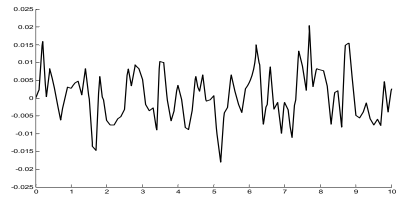


Fig.7 . disturbance added to detune friction from its nominal level.

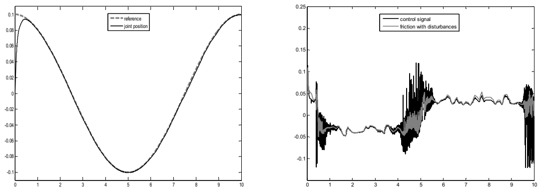


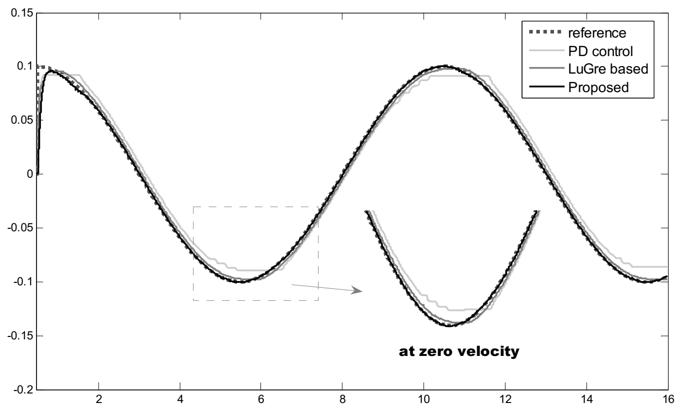
Fig.8 . Joint position, proposed control with disturbance

The results after compensation shown in fig.8 demonstrate the robustness of the proposed method.

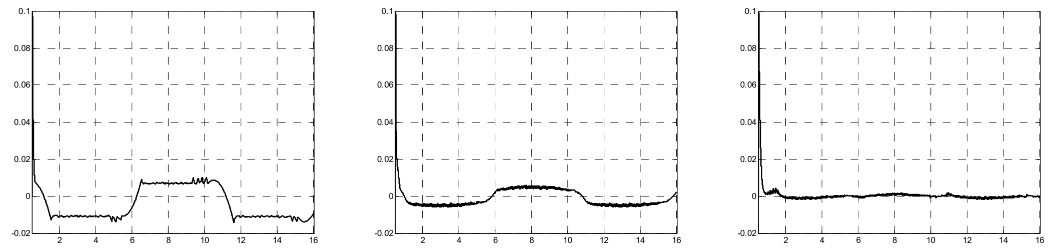
The experimental setup consists of a 700 MHz PC operating under RT-LINUX and a digital servo adapter which communicate via an optical cable to ensure signal denoising [11]. The control algorithms are implemented in C using a sample time of $T_s = 0.001$s.

The performance of the proposed method is now evaluated in simulations and experimental environments. The LuGre model (l_f = 0) is used to generate real friction forces in the simulations whereas the model-based observer is used in our experiments for comparison [13];

$$\begin{aligned} \dot{z} &= v - \frac{c_0}{g(v)}|v|z - l_f.u^* \\ F &= c_0 z + \sigma_1 \dot{z} + F_v v \end{aligned} \tag{12}$$



a. Joint tracking position (f =0.1Hz), Experimental results comparison.



b. Tracking error (f =0.1Hz), Experimental results comparison: PD control (left), LuGre-based (middle), proposed(right).

Fig.9. Robot joint tracking 0.1sin(0.2 πt) (rad). Experimental results.

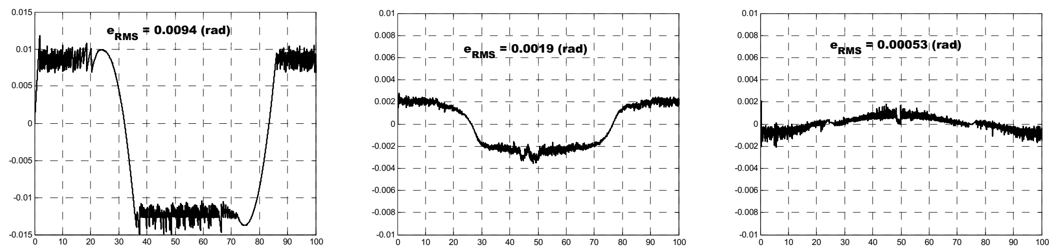


Fig.10. Robot joint tracking 0.1sin(0.02 πt) (rad). Tracking error (f =0.01Hz), Experimental results comparison: PD control (left), LuGre-based (middle), proposed (right)..

Where $g(v) = F_C + (F_S - F_C)e^{v^2/v_s^2}$ is a Gaussian function with all parameters defined in Table I. This function is used to fit the steady state friction curve. It should be noted that it is very difficult to reproduce accurately the Stribeck friction curve with $g(v)$.

Fig. 9 shows experimental result of the robot joint tracking a sine reference for f =0.1Hz, in this case the maximum value of reference velocity is can reach 0.0628rad/s, slow motions where the level of friction is very hard to define accurately, and the proposed approach is more effective to eliminate friction induced errors in two stages: first, the major part is compensated by the minor loop, and the tracking error coming from uncertain compensation is minimized by the gain scheduled controller. For slower motions (fig.10), the calculated RMS error clearly indicates the effectiveness of the proposed method $e_{RMS} = 0.00053 rad$ over PD control case $e_{RMS} = 0.0094 rad$ and PD+LuGre based compensator $e_{RMS} = 0.0019 rad$.

V. Conclusion

A dynamic friction model structure has been proposed to the problem of tracking control under uncertain friction compensation in slow motion regime. Motivated by the dynamic nature of friction, the estimation mechanism uses local properties and ensures in a minor loop a part of the control signal to cancel friction effects at very low velocities. Under inexact compensation, a high performance robust gain-scheduled controller is synthesized. This approach can be the basis of further development, using the relatively simple structure of the local models and allowing methods like robust adaptive control and hybrid-based control strategies to be applied to a variety of motion systems such as: machine tools, robots, hard disc drives and even in vehicle stability enhancement where a tire-road friction model is needed.

Appendix

Table 1 simulation and design parameters.

F_S (N.m)	0.075
F_c (N.m)	0.045
F_v	0.056
v_s (m/s)	0.1
σ_0	950
σ_1	1.0
m	0.001
K_P	6.0
K_D	2.0
K	0.1
v_i (rad/s)	-1,-0.05,-0.001,0,0.001,0.05,1
K_i	0,-0.5,-1.5,-4.5,-1.5,0.5,0

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