NMCDA: A Framework for Evaluating Cloud Computing Services

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Abstract

Many organizations are currently seeking to contract services from cloud computing rather than owning the possessions to supply those services. Due to the fast development of cloud computing, many cloud services have been developed. Any organization that tries to achieve the best flexibility and quick response to market requests, they have the options to use cloud services. Due to the variety of cloud service providers, it is a very significant challenge for organizations to select the appropriate cloud services which can fulfill their requirements, as many criteria should be considered in the selection process of cloud services. Therefore, the selection process of cloud services can be considered as a type of multi-criteria decision analysis problems. In this research paper, we present how to aid a decision maker to estimate different cloud services by providing a neutrosophic multi-criteria decision analysis (NMCDA) approach for estimating the quality of cloud services. Triangular neutrosophic numbers are used to deal with ambiguous and inconsistent information which exist usually in the performance estimation process. An efficacious model is evolved depending on neutrosophic analytic hierarchy process (NAHP). The aim is to solve the performance estimation problem and improve the quality of services by creating a strong competition between cloud providers. To demonstrate the pertinence of the proposed model for disbanding the multi-criteria decision analysis, a case study is presented.

Keywords Neutrosophic Analytic Hierarchy Process (NAHP), Cloud Services, Neutrosophic Set, Triangular Neutrosophic Numbers, Multi-Criteria Decision Analysis (MCDA), Consistency.

1. Introduction

Cloud computing turned into a prevalent service due to the fast evolution of information and communication technologies [1]. Clouds are computing and data-
storage systems which integrate different technologies together to interconnect and manage demanded resources on dispensed computers [2]. The conceptual view of cloud computing is presented in Fig.1.

Figure 1. Cloud computing

Several definitions of Cloud computing have been determined in different methods from analyst corporations, academics, manufacture practitioners, and IT firms. Table 1 presents various definitions of cloud computing according to many analyst corporations.

Table 1. Definitions of Cloud Computing according to various analyst firms

<table>
<thead>
<tr>
<th>Analyst corporations</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gartner</td>
<td>“A computing technique in which a huge scalable IT linked capabilities are introduced “as a service” via utilizing Internet to various outer users” [3]. “A development, deployment and delivery IT model for transmitting services, products and solutions through the Internet” [4].</td>
</tr>
<tr>
<td>IDC</td>
<td>“A service model which integrates a generic arrangement standard for IT transmission, infrastructure ingredients, an architectural path and an economic model – essentially, a concourse of grid computing, virtualization, utility-</td>
</tr>
</tbody>
</table>
The applications of cloud computing [7] are as follows:

1. Secure and dependable center of data storage.
2. Share data between various equipments.
3. Enables users to use the internet infinitely.
4. Does not require high quality equipment form user.

There exist three layers (delivery models) of cloud computing [8]:

1. The first layer is Software as a Service (SaaS), which extends access service to whole applications.
2. The second layer is Platform as a Service (PaaS), which extends a platform for improving other applications on its head.
3. The third layer is Infrastructure as a Service (IaaS), which run, prevail and manage storages and virtual machines through the extended environment.

The delivery services on three layers: (IaaS), (PaaS), and (SaaS), concerned by cloud computing. By introducing interfaces on all three layers, clouds declaim various kinds of customers:

1. End consumers, who basically use the services of the SaaS layer via a Web browser and requisite offerings of the IaaS layer.
2. Business customers, who can access all layers: to improve the own infrastructure with additional resources on demand they access the IaaS layer, to be able to run own applications in a cloud they access the PaaS layer and they access the SaaS layer to gather benefits of available applications which presented as a service.
3. Developers and Independent Software Vendors, who evolves applications which are given through the SaaS layer of a cloud.

The different types of cloud deployment models are as follows [8, 9]:

1. Public clouds: In these types, the service providers present their services to the public through the internet and web applications. Public clouds need a high level of security and control to manage business situations effectively.
2. Private clouds: These types are established especially for single organization.
These types of clouds are characterized by a high level of security and control, because only stakeholders of organization have access to it. Private cloud is very expensive by comparing it with public clouds.

3. Community cloud: This type is established between two or more organizations that have the same requirements.

4. Hybrid clouds: A combination between at least two clouds. To keep some data in an organization, a hybrid cloud support this feature.

5. Virtual private cloud: To overcome the drawbacks of private and public clouds, a virtual private cloud is presented.

The models and characteristics of cloud computing presented in Fig. 2.

Clouds are very useful for organizations due to the following reasons:

1. Inspire and preserve the competitive features of organizations.
2. Ameliorate the organization performance in the marketplace.
3. Better management of organizational information systems and reducing costs.
4. Increase the productivity of the organization.
5. Improve cooperation among organization members.
6. Create a flexible environment among organization members.[10]
7. Reduce cost of organization.

The famous criteria for the estimation process of cloud services as mentioned in many researches are as follows:

1. Security: Is the ability to keep data for organization and achieving confidence and privacy by the cloud service.
2. Performance: Is the quality of service, which provided by the cloud service providers.
3. Accessibility and usability: The ease of use of cloud service for supporting organization requirements.
4. Scalability: Is the ability of cloud service to fit the problem and use resources effectively.
5. Adaptability: The adjustment process of cloud services depending on customer requests.

Once the previous criteria have been identified, the cloud services have to be estimated by a multi-criteria decision analysis approach (MCDA).

MCDA is a group of theories, methodologies and techniques for transacting with different problems. In decision making problems, the MCDA approaches choose and rank the actions efficiently. MCDA, is a significant branch in operations research, seeks to plan mathematical and programming tools to select the superior alternative between various choices, according to particular criteria. The MCDA approaches are categorized into two types in [11]:

1. Multi-Attribute Utility Theory (MAUT): seeks to get a function which reflect the utility of a specific alternative. Each alternative assigns a marginal utility, with a real number presenting the prefer-ability of alternative. The final utility is the sum of these marginal utilities.

2. Outranking approaches: construct a pairwise comparison matrices to determine whether one alternative is ranked greater than another.

The MCDA approaches and its capabilities presented in table 2.
<table>
<thead>
<tr>
<th>Technique</th>
<th>Capabilities</th>
<th>Authors/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal programming</td>
<td>Solve multiple and conflicting problems and it's an application of linear programming [12]</td>
<td>Charnes et al. (1955)</td>
</tr>
<tr>
<td>DEMATEL</td>
<td>used to rank compromises a structural model including associations of ganglion factors [14]</td>
<td>Gabus and Fontela (1973)</td>
</tr>
<tr>
<td>ANP</td>
<td>a generalization of AHP, represent the interrelationships between decision levels and attributes [16]</td>
<td>Thomas L. Saaty (1996)</td>
</tr>
<tr>
<td>DEA</td>
<td>Relative to a set of similar observations it evaluate the competence of an observation [17]</td>
<td>Charnes (1978)</td>
</tr>
<tr>
<td>ELECTRE</td>
<td>determine and exclude alternatives which are dominated by other alternatives [18]</td>
<td>Roy (1991)</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>Select the alternative which is adjacent to the positive ideal solution and</td>
<td>Hwang and Yoon (1981)</td>
</tr>
</tbody>
</table>
Zadeh, proposed fuzzy set theory in 1965 [22], and it has been widely used to handle the ambiguous of human's decision [23]. It also able to resolve uncertainties which exist in information for MCDM/ MCDA. In fuzzy MCDA, the important weights of criteria are estimated using linguistic values represented via fuzzy numbers. Since fuzzy set theory, considers only the truth-membership degree, it fails to represent reality and can't represent vague and inconsistent information efficiently. In order to enhance performance of fuzzy set theory, Atanassove introduced intuitionistic fuzzy sets [24]. It can only model incomplete information and cannot model indeterminacy and inconsistency which exists usually in real systems. In 1995, Smarandache introduced Neutrosophy [25]. Neutrosophic set is a popularization of classic set, fuzzy set, and intuitionistic fuzzy set, etc. To facilitate the practical side of neutrosophic sets, a single-valued neutrosophic set (SVNS) was presented [26]. It consists of three membership degrees which are the truth, indeterminacy and falsity degrees. In neutrosophic set indeterminacy is explicitly quantified, truth, indeterminacy and falsity membership functions are independent. This proposition is very significant in various status such as information fusion when we try to integrate the data from various sensors. The single valued neutrosophic set applied in various domains [27-29]. When decision maker gives his/her opinion about a statement, he/she may say that, this statement is 50% true, 60% false and 20% I am not sure, so we can say that neutrosophic is the best concept for representing real decision making process via considering truth (sureness), indeterminacy (not sure) and falsity (false) membership functions. It can also manage vague, incomplete and inconsistent information efficiently. Nowadays it has an extensive application in various domains [30-33].

The estimation process of available cloud services is a complex problem due to the following reasons [34]:

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRA</td>
<td>Solve problems with complex interrelationships among factors and variables</td>
<td>Deng (1982)</td>
</tr>
<tr>
<td>PROMETHEE</td>
<td>As ELECTRE, but various in the pairwise comparison phase</td>
<td>Brans and Vincke (1986)</td>
</tr>
</tbody>
</table>
1. Numerous and incompatible criteria.
2. Different interests of decision makers.
3. The diversity of cloud services.
4. Failure to handle vague and inconsistent information which exist usually in the performance estimation process.
5. The altitude cognitive request on the decision makers [35-37].

Since the estimation process of available cloud services is a complex problem, then it can be solved using analytic hierarchy process (AHP) through converting complex problems to sub-problems for solving it easier. The hierarchal structure of AHP consist of goal in the top level, followed by criteria and sub-criteria and at the final level exist alternatives. This hierarchal structure of AHP helps decision makers to understand and solve problem easier. But in classical AHP the judgments of experts takes crisp values and this is not accurate in reality due to vague, inconsistent information. So to overcome the previous drawbacks of estimation process of cloud services via using AHP, a multi-criteria decision analysis approach depending on the analytic hierarchy process in neutrosophic environment presented in this research. Using MCDA (AHP) in neutrosophic environment represents truth, indeterminate and falsity degrees efficiently. Then it makes a real and accurate decision via making a precise judgments using neutrosophic numbers. The proposed model also aggregates different interests of decision makers into one opinion to delete confliction, and it treats inconsistency of experts judgments and improve consistency degree using the neutrosophic induced bias matrix. The selected cloud service providers via using proposed model, will be the suitable and accurate selection of decision makers. A case study is solved to explain the pertinence of the proposed model.

This research is structured as follows:
Section 2 illustrates the literature review about choosing cloud service providers using various MCDA methods. Section 3 illustrates the basic definitions of neutrosophic sets. Section 4 presents the proposed model of neutrosophic multi-criteria decision analysis approach depending on the analytic hierarchy process. Section 5 validates the model by solving a case study. Section 6 compares the proposed model with other existing models and evaluates it. Section 7 concludes the research and determines the future directions of the work.
2. Literature Review

In this section a survey on various MCDA techniques which are used in selection process of cloud service.

Decision makers in many organizations face a major challenge in choosing and estimating the most suitable requirements [1, 10] of cloud. Implementing cloud services in an organization and estimating their performance is a complex process due to the following reasons:

1. Incompatible and numerous criteria.
2. Different interests of decision makers.
3. Imprecision latent in the estimation process.

Due to the variety of cloud service providers, it is a very significant defy for organizations to select the appropriate cloud providers which can fulfill their needs. The criteria for estimating and selecting the desirable cloud services, should be determined first. The pertinent criteria for estimating the performance of cloud services have been identified in many researches[1,8,38-51].

The estimation and selection process of cloud services has been illustrated in many researches by using various methods [8, 38, 39]. The analytic hierarchy process has been used by Garg et al. [8] to estimate the effectiveness of cloud services in an organization. In a multi-sourcing scenario, a mathematical decision making pattern for selecting cloud services proposed in [52]. The AHP applied to task-oriented resource allocation of cloud computing in [53]. A new AHP of cloud service selection applied to medical service cloud environment proposed in [54]. The AHP is an effective and efficient decision-making technique but subjectivity of decision makers can yield uncertainties when performing pairwise comparisons. To overcome this drawback, fuzzy AHP has been used by Safari et al. [55] for prioritizing cloud computing acceptance indicators. The fuzzy AHP used by Singla and Kaushal [56] for cloud path selection of offloading in mobile cloud computing. Also fuzzy AHP approach has been used by Cheng [57] for cloud computing decision making problem. It is often hard and not accurate for decision makers to exactly determine his or her opinion within the interval [0, 1]. Hence, interval valued fuzzy AHP is proposed in [58]. The fuzzy set theory can be applied to various decision making problems via possess a degree of uncertainty, but the resulted judgment is always somewhat vague. A novel MCDA method proposed in [59] to assess cloud computing service using intuitionistic
fuzzy sets. We are the first to propose a MCDA technique based on AHP using neutrosophic sets. The simple additive weighting (SAW) approach has been used by Saripalli and Pingali[38] for transacting with the cloud service estimation and selection problem. The analytic network process (ANP) integrated with zero-one goal programming by Menzel et al.[39] to evaluate the quality of cloud services. A fuzzy multi-criteria group decision making technique based on the technique for order preference by similarity to the ideal solution (TOPSIS) has been used by Wibowo et al. [60] for evaluating cloud services.

A new hybrid fuzzy multicriteria group decision-making approach, depend on integration of fuzzy set and modified VIKOR techniques, was presented in [36]. A decision making model which combine interval-valued fuzzy sets and VIKOR is proposed in [61] for evaluating and selecting suitable cloud service provider.

3. Concepts and Definitions of Neutrosophic Set

The requisite definitions of neutrosophic sets, triangular neutrosophic numbers and its operations presented in this section [25,26].

**Definition 1.** Any neutrosophic set $A$ in $X$ has a truth $T_A(x)$, indeterminacy $I_A(x)$ and falsity $F_A(x)$ membership functions. Where $X$ is a set of points, $x \in X$, $T_A(x):X \rightarrow [0,1]$, $I_A(x):X \rightarrow [0,1]$ and $F_A(x):X \rightarrow [0,1]$. The sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ has no restriction.

**Definition 2.** A single valued neutrosophic set $A$ over $X$ is an object having the form $A=\{(x,T_A(x), I_A(x), F_A(x)): x \in X\}$, where $T_A(x):X \rightarrow [0,1]$, $I_A(x):X \rightarrow [0,1]$ and $F_A(x):X \rightarrow [0,1]$ with $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$ for all $x \in X$. For convenience, a SVN number is represented by $A=(a, b, c)$, where $a, b, c \in [0,1]$ and $a+b+c \leq 3$.

**Definition 3.** Suppose that $\alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a} \in [0,1]$ and $a_1, a_2, a_3 \in R$ where $a_1 \leq a_2 \leq a_3$. Then a single valued triangular neutrosophic number, $\tilde{a}=(a_1, a_2, a_3); \alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a}$ is a neutrosophic set whose truth, indeterminacy and falsity membership functions are as follows:

$$T_{\tilde{a}}(x) = \begin{cases} 
\alpha_{\tilde{a}} \frac{x-a_1}{a_2-a_1} & (a_1 \leq x \leq a_2) \\
\alpha_{\tilde{a}} & (x = a_2) \\
\alpha_{\tilde{a}} \frac{a_3-x}{a_3-a_2} & (a_2 < x \leq a_3) \\
0 & \text{otherwise} 
\end{cases} \quad (1)$$
\[ I_\tilde{a}(x) = \begin{cases} \frac{(a_2 - x + \theta_\tilde{a}(x - a_1))}{(a_2 - a_1)} & (a_1 \leq x \leq a_2) \\ \theta_\tilde{a} & (x = a_2) \\ \frac{(x - a_2 + \theta_\tilde{a}(a_3 - x))}{(a_3 - a_2)} & (a_2 < x \leq a_3) \\ 1 & \text{otherwise}, \end{cases} \]

\[ F_\tilde{a}(x) = \begin{cases} \frac{(a_2 - x + \beta_\tilde{a}(x - a_1))}{(a_2 - a_1)} & (a_1 \leq x \leq a_2) \\ \beta_\tilde{a} & (x = a_2) \\ \frac{(x - a_2 + \beta_\tilde{a}(a_3 - x))}{(a_3 - a_2)} & (a_2 < x \leq a_3) \\ 1 & \text{otherwise}. \end{cases} \]

Where \( \alpha_\tilde{a}, \theta_\tilde{a} \) and \( \beta_\tilde{a} \), represent the greatest degree of truth membership, least degree of indeterminacy and falsity memberships respectively.

**Definition 4.** Let \( \tilde{a} = ((a_1, a_2, a_3); \alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a}) \) and \( \tilde{b} = ((b_1, b_2, b_3); \alpha_\tilde{b}, \theta_\tilde{b}, \beta_\tilde{b}) \) be two single valued triangular neutrosophic numbers and \( \gamma \neq 0 \) be any real number. Then,

1. **Addition of two triangular neutrosophic numbers**
   \[ \tilde{a} + \tilde{b} = ((a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_\tilde{a} \land \alpha_\tilde{b}, \theta_\tilde{a} \lor \theta_\tilde{b}, \beta_\tilde{a} \lor \beta_\tilde{b}) \]

2. **Subtraction of two triangular neutrosophic numbers**
   \[ \tilde{a} - \tilde{b} = ((a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_\tilde{a} \land \alpha_\tilde{b}, \theta_\tilde{a} \lor \theta_\tilde{b}, \beta_\tilde{a} \lor \beta_\tilde{b}) \]

3. **Inverse of a triangular neutrosophic number**
   \[ \tilde{a}^{-1} = \left( \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}; \alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a} \right), \text{Where}(\tilde{a} \neq 0) \]

4. **Multiplication of triangular neutrosophic number by constant value**
   \[ \gamma \tilde{a} = \left\{ \begin{cases} ((\gamma a_1, \gamma a_2, \gamma a_3); \alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a}) & \text{if } (\gamma > 0) \\ ((\gamma a_3, \gamma a_2, \gamma a_1); \alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a}) & \text{if } (\gamma < 0) \end{cases} \right. \]

5. **Division of triangular neutrosophic number by constant value**
   \[ \frac{\tilde{a}}{\gamma} = \left\{ \begin{cases} \left( \frac{a_1}{\gamma}, \frac{a_2}{\gamma}, \frac{a_3}{\gamma}; \alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a} \right) & \text{if } (\gamma > 0) \\ \left( \frac{a_3}{\gamma}, \frac{a_2}{\gamma}, \frac{a_1}{\gamma}; \alpha_\tilde{a}, \theta_\tilde{a}, \beta_\tilde{a} \right) & \text{if } (\gamma < 0) \end{cases} \right. \]

6. **Division of two triangular neutrosophic numbers**
The steps of the proposed approach are as follows:

**Step 1:** Draw the hierarchy of the problem at various levels, which is called the decomposition process.

**Step 2:** Let decision makers compare criteria and alternatives through the linguistic terms, which shown in table 3 and represented according to Abdel-Basset opinion.
### Table 3. Linguistic terms and the identical triangular neutrosophic numbers

<table>
<thead>
<tr>
<th>Saaty scale</th>
<th>Explanation</th>
<th>Neutrosophic Triangular Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally significant</td>
<td>$\tilde{1} = \langle (1,1,1); 0.50,0.50,0.50 \rangle$</td>
</tr>
<tr>
<td>3</td>
<td>Slightly significant</td>
<td>$\tilde{3} = \langle (2,3,4); 0.30,0.75,0.70 \rangle$</td>
</tr>
<tr>
<td>5</td>
<td>Strongly significant</td>
<td>$\tilde{5} = \langle (4,5,6); 0.80,0.15,0.20 \rangle$</td>
</tr>
<tr>
<td>7</td>
<td>very strongly significant</td>
<td>$\tilde{7} = \langle (6,7,8); 0.90,0.10,0.10 \rangle$</td>
</tr>
<tr>
<td>9</td>
<td>Absolutely significant</td>
<td>$\tilde{9} = \langle (9,9,9); 1.00,0.00,0.00 \rangle$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>$\tilde{2} = \langle (1,2,3); 0.40,0.65,0.60 \rangle$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>$\tilde{4} = \langle (3,4,5); 0.60,0.35,0.40 \rangle$</td>
</tr>
<tr>
<td>6</td>
<td>sporadic values between two close scales</td>
<td>$\tilde{6} = \langle (5,6,7); 0.70,0.25,0.30 \rangle$</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>$\tilde{8} = \langle (7,8,9); 0.85,0.10,0.15 \rangle$</td>
</tr>
</tbody>
</table>

If the decision maker illustrates "criterion 1 is absolutely significant than criterion 2", then it takes the triangular neutrosophic scale as $\langle (9,9,9); 1.00,0.00,0.00 \rangle$. Conversely, the comparison of criterion 2 to criterion 1 will take the triangular neutrosophic scale as $\langle (1/9, 1/9, 1/9); 1.00,0.00,0.00 \rangle$.

The pair-wise comparison matrices will have the following form as shown in Eq.4:

$$
\tilde{A}^k = \begin{bmatrix}
\tilde{r}_{11}^k & \tilde{r}_{12}^k & \cdots & \tilde{r}_{1n}^k \\
\tilde{r}_{n1}^k & \tilde{r}_{n2}^k & \cdots & \tilde{r}_{nn}^k
\end{bmatrix}
$$

(4)

Where $\tilde{r}_{ij}^k$ is the preference relation of $i^{th}$ criterion over $j^{th}$ criterion according to $k^{th}$ decision maker. The "tilde" symbolize the triangular neutrosophic numbers, which have the following form $\tilde{r}_{ij}^k = \langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \rangle$, where $l_{ij}, m_{ij}, u_{ij}$ are the lower, median and upper bound of neutrosophic number, $T_{ij}, I_{ij}, F_{ij}$ are the truth-membership, indeterminacy and falsity membership functions respectively of triangular neutrosophic number. For example $\tilde{r}_{12}^1$ is the preference relation of first criterion via second criterion, with respect to the first decision makers and equal to $\tilde{r}_{12}^1 = \langle (2,3,4); 0.30,0.75,0.70 \rangle$. Here in this research the truth, indeterminacy and falsity membership functions quantified for each triangular neutrosophic number according to decision maker opinion.

**Step 3:** By having more than on decision maker in the estimation process then, the aggregated $\tilde{r}_{ij}$ of all decision makers calculated as in Eq.5 for obtaining the final
comprehensive preference values via taking average values of all decision makers preferences.

\[
\tilde{r}_{ij} = \frac{\sum_{k=1}^{\kappa}((l_{ij}^k, m_{ij}^k, u_{ij}^k); T_{ij}^k, I_{ij}^k, F_{ij}^k)}{\kappa}
\]  

(5)

The aggregated pair-wise comparison matrix according to the averaged preference values has the following form:

\[
\tilde{A} = \begin{bmatrix}
\tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n} \\
\vdots & \ddots & \vdots & \vdots \\
\tilde{r}_{n1} & \tilde{r}_{n2} & \cdots & \tilde{r}_{nn}
\end{bmatrix}
\]  

(6)

**Step 4:** from the previous matrix we can calculate weight and creating a ranking of priorities, as follows:

1. Take the totality row averages:

\[
\tilde{w}_i = \frac{\sum_{j=1}^{n}((l_{ij}^k, m_{ij}^k, u_{ij}^k); T_{ij}^k, I_{ij}^k, F_{ij}^k)}{n}
\]  

(7)

2. Since \(\tilde{w}_i\) are still triangular neutrosophic numbers, so we need to de-neutrosophic it using the following score equation:

\[
S(\tilde{r}_{ij}) = \left|\frac{(l_{ij} + m_{ij} + u_{ij})}{3} + (T_{ij} - I_{ij} - F_{ij})\right|
\]  

(8)

\[
S(\tilde{w}_i) = \left|\frac{(l_{wi} + m_{wi} + u_{wi})}{3} + (T_{wi} - I_{wi} - F_{wi})\right|
\]  

(9)

The previous score function apply to each triangular neutrosophic number for converting it to its crisp numerical value via taking the mean value of triangular number and added it to confirmation degree which equal \((T - I - F)\) of triangular number.

After de-neutrosophic of \(\tilde{w}_i\), we obtain \(w_i\),which is a crisp value and we need to normalize it using the following equation:

\[
w_i^N = \frac{w_i}{\sum_{i=1}^{n}w_i}
\]  

(10)

**Step 5:** Check consistency of judgments.

To ensure decision quality, we have to consider the consistency of the pair-wise comparison matrix during the evaluation process. If the pair-wise comparison matrix has a transitive relation i.e. \(a_{ik} = a_{ij}a_{jk}\) for all \(i, j\) and \(k\), then the comparison matrix is consistent. But this method doesn't calculate the degree of consistency or inconsistency (i.e. the greater or lesser degree of consistency and inconsistency). So in this research we use the transitive relation (i.e. \((l_{ik}, m_{ik}, u_{ik})\)
=\left( l_{ij}, m_{ij}, u_{ij} \right), \left( l_{jk}, m_{jk}, u_{jk} \right)) to determine the consistency and calculate consistency degree according to Saaty. Not only this, but we also enhance the degree of consistency for the pair-wise comparison matrix and make it consistent by developing the concept in [62]. The value of the consistency of the pair-wise comparison matrix depend on \( n \) (i.e. the number of items being compared), and the consistency rate (CR) have to be calculated. The consistency rate is the ratio between the consistency index (CI) and a random consistency index (RI). The value of (CR) shouldn't exceed 0.1 for comparison matrix which is smaller than or equal to \( 4 \times 4 \). The pair-wise comparison matrix is convenient, if the upper-bound of the consistency rate like what is shown in table 4 [63, 64].

**Table 4. Upper bound for pair-wise comparison matrix to be convenient**

<table>
<thead>
<tr>
<th>N</th>
<th>3 × 3</th>
<th>4 × 4</th>
<th>n&gt;4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR≤</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
</tr>
</tbody>
</table>

For calculating CI and CR, the following steps should be executed:

1. All values in the first column of the pair-wise comparison matrix should be multiplied by the priority of the first item; this process continues for all columns of the comparison matrix. Sum the values across the rows to obtain a vector of values labeled "weighted sum".
2. The elements of the weighted sum vector should be divided by the corresponding priority for each criterion.
3. Compute the mean of the values found in the previous step; this mean is denoted \( \tilde{\lambda}_{\text{max}} \).
   Since \( \tilde{\lambda}_{\text{max}} \) still neutrosophic number, then we need to de-neutrosophic it \( \left( \lambda_{\text{max}} \right) \) by using in Eq.8 as follows:

\[
S(\tilde{\lambda}_{\text{max}}) = \left| \left( l_{\tilde{\lambda}_{\text{max}}} + m_{\tilde{\lambda}_{\text{max}}} + u_{\tilde{\lambda}_{\text{max}}} \right)/3 + (T_{\tilde{\lambda}_{\text{max}}} - l_{\tilde{\lambda}_{\text{max}}} - F_{\tilde{\lambda}_{\text{max}}}) \right| (11)
\]

4. Compute the consistency index (CI) as follows:

\[
\text{CI} = \frac{\tilde{\lambda}_{\text{max}} - n}{n-1} (12)
\], Where \( n \) is the number of criteria being compared.
5. Compute the consistency ratio, which is defined as:
\( CR = \frac{CI}{RI} \) \quad (13)

Where RI is the consistency index of a randomly generated pair-wise comparison matrix and shown in table 5.

**Table 5.** Saaty table for random consistency index (RI) per different number of criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>0.0</td>
<td>0.0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.40</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

**Step 6:** If the pair-wise comparison matrix isn’t consistent, the decision maker doesn’t repeat the exercise as in classical AHP but he/she can repair the matrix and make it consistent and he/she can also enhance the degree of consistency by using the following steps:

To identify the inconsistent elements in a pair-wise comparison matrix or enhance the consistency degree, Ergu et al.[62] proposed the induced bias matrix. The theorem and corollaries are as follows:

**Theorem 1:** If the comparison matrix \( \tilde{A} \) is a consistent matrix then, neutrosophic induced matrix \( \tilde{I} = \tilde{A} \times \tilde{A} - n \times \tilde{A}. \)

**Corollary 1:** If the comparison matrix \( \tilde{A} \) is approximately consistent, then the neutrosophic induced matrix \( \tilde{I} \) should be close to zero.

**Corollary 2:** If the pair-wise matrix is inconsistent, then there must exist some inconsistent element.

Ergu et al.[62] proposed major steps to identify inconsistency, and we developed these steps to deal with neutrosophic theory and enhance consistency degree of pair-wise matrix as follows:

1. Construct the neutrosophic induced matrix \( \tilde{I} = \tilde{A} \times \tilde{A} - n \times \tilde{A}. \)
2. Determine the largest preference relation \( \tilde{r}_{ij} \), which has the largest lower, median and upper-bound of triangular number.
3. Determine the \( i^{th} \) row and \( j^{th} \) column which contain the inconsistent triangular neutrosophic number and calculate the dot product of row vector \( \tilde{R}_i = (\tilde{r}_{i1}, \tilde{r}_{i2}, ..., \tilde{r}_{in}) \) and column vector \( \tilde{C}_j^T = (\tilde{r}_{1j}, \tilde{r}_{2j}, ..., \tilde{r}_{nj})^T \), where \( \tilde{C}_j^T \) is the transpose vector of \( \tilde{C}_j \).
4. The dot product \( \tilde{P} = \tilde{R}_i \cdot \tilde{C}_j^T = (\tilde{r}_{11} \tilde{r}_{1j}, \tilde{r}_{12} \tilde{r}_{2j}, ..., \tilde{r}_{in} \tilde{r}_{nj}) \) \quad (15)
5. Calculate elements which are distant from $\tilde{r}_{ij}$ in vector $\tilde{P}$ by the following formula: $\tilde{b} = \tilde{P} - \tilde{r}_{ij} (16)$, where $\tilde{P}$ is the prejudice vector.

6. Determine the elements in the original pair-wise comparison matrix $\tilde{A}$ that cause inconsistency, by using the prejudice vector.

7. These elements are with the largest lower, median and upper bounds and far from scratch in the prejudice vector.

8. Try to modify these elements for enhancing the consistency of the judgments.

**Step 7:** Calculate the normalized weight of alternatives as in criteria weight calculation process.

**Step 8:** To calculate the scores of alternatives, multiply the weight of each alternative with the related criteria.

**Step 9:** Rank alternatives according to the largest score value.

The overall description of the proposed model presented in Fig.3.
Figure 3. Model description.

5. Case Study: Ranking of Cloud Computing Based on Proposed Model

The proposed neutrosophic multi-criteria decision analysis approach submitted in
section 4 is utilized for estimating cloud computing services quality for the next case study.

A big e-learning services provider company in Egypt contains greater than 100 employees. The company has major activities such as e-learning content expansion and business delivery over frontal marketing. The top regards of the company are safeness and piracy of the contents. If the contents of the company are pirated on the internet, this will cause wasteful loss, and for this reason the company is looking for the appropriate cloud service. The board of directors of the company nominated three cloud service alternatives, according to five of the most important criteria in the estimation process. The cloud computing service alternatives are (1) Dropbox, (2) Google Drive and (3) Microsoft Sky Drive. The five criteria in the cloud service estimation process are (1) Security, (2) Performance, (3) Accessibility, (4) Scability and (5) Adaptability.

**Step 1:** Draw the hierarchy of cloud service estimation process as in Fig.4.

![Cloud service performance estimation process](image)

**Figure 4.** The hierarchical structure of criteria and alternatives

The decision makers here are IT administrators and the information about them are presented in table 6.

**Table 6.** Record for interview

<table>
<thead>
<tr>
<th>Biographical characteristics about job</th>
<th>Interviewers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job</td>
<td>IT managers</td>
</tr>
<tr>
<td>Sector</td>
<td>Service and sales</td>
</tr>
</tbody>
</table>
Step 2: For allowing decision makers to compare criteria and alternatives through the linguistic terms shown in table 3, a meeting was executed with the directors of the company and the averaged preference relation of the criteria is presented in table 7, where $C_1, \ldots, C_5$ are the criteria’s names as listed in the main example respectively.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>$((1, 1, 1):0.50,0.50,0.50)$</td>
<td>$((1, 1, 1):0.50,0.50,0.50)$</td>
<td>$((4, 5, 6):0.80,0.15,0.20)$</td>
<td>$((6, 7, 8):0.90,0.10,0.10)$</td>
<td>$((4, 5, 6):0.80,0.15,0.20)$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$((1, 1, 1):0.50,0.50,0.50)$</td>
<td>$((1, 1, 1):0.50,0.50,0.50)$</td>
<td>$((4, 5, 6):0.80,0.15,0.20)$</td>
<td>$((6, 7, 8):0.90,0.10,0.10)$</td>
<td>$((6, 7, 8):0.90,0.10,0.10)$</td>
</tr>
<tr>
<td>$c_3$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.80,0.15,0.20)$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.80,0.15,0.20)$</td>
<td>$((1, 1, 1):0.50,0.50,0.50)$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.30,0.75,0.70)$</td>
<td>$((2, 3, 4):0.30,0.75,0.70)$</td>
</tr>
<tr>
<td>$c_4$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.90,0.10,0.10)$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.90,0.10,0.10)$</td>
<td>$((2, 3, 4):0.30,0.75,0.70)$</td>
<td>$((1, 1, 1):0.50,0.50,0.50)$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.80,0.15,0.20)$</td>
</tr>
<tr>
<td>$c_5$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.80,0.15,0.20)$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.80,0.15,0.20)$</td>
<td>$((\frac{1}{6}, \frac{5}{6}, \frac{4}{6}):0.30,0.75,0.70)$</td>
<td>$((4, 5, 6):0.80,0.15,0.20)$</td>
<td>$((1, 1, 1):0.50,0.50,0.50)$</td>
</tr>
</tbody>
</table>

Step 3: Calculating weights of criteria as follows:

1. Take the totality row averages using the following equation:
   \[ \widehat{w}_i = \frac{\sum_{j=1}^{n} (t_{ij}, m_{ij}, u_{ij})}{n} \]
   Then, \[ \widehat{w}_1 = ((3.2, 3.8, 4.4); 0.50, 0.50, 0.50), \]
   \[ \widehat{w}_2 = ((3.6, 4.2, 4.8); 0.50, 0.50, 0.50), \]
   \[ \widehat{w}_3 = ((0.7, 0.95, 1.2); 0.30, 0.75, 0.70), \]
   \[ \widehat{w}_4 = ((0.68, 0.89, 1); 0.30, 0.75, 0.70), \]
   \[ \widehat{w}_5 = ((1.1, 1.3, 1.6); 0.30, 0.75, 0.70). \]

2. Since \( \widehat{w}_i \) are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq.9.
   \[ w_1 = 3.3, w_2 = 3.7, w_3 = 0.2, w_4 = 0.29, w_5 = 0.18 . \]

3. After de-neutrosophic \( \widehat{w}_i \) , we obtain \( w_i \), which is a crisp value and we need to normalize it using Eq.10.

Then, the normalized weight value of criteria are as follows:

\[ w_1 = 0.4, w_2 = 0.5, w_3 = 0.03, w_4 = 0.04, w_5 = 0.02. \]

It's obvious that \( \sum w_i = 1. \)
The priorities of criteria are as follows: 
C₂, C₁, C₄, C₃ and C₅ respectively.

**Step 4:** Check consistency of judgments.
If the pair-wise comparison matrix has a transitive relation i.e.\( a_{ik} = a_{ij}a_{jk} \) for all \( i, j \) and \( k \), then the comparison matrix is consistent. By focusing only on the lower, median and upper values of triangular neutrosophic number of the comparison matrix of criteria it's obvious that the matrix is consistent. To measure the degree of consistency do the following steps:

1. Calculate the "weighted sum" for each row.
   The weighted sum of the first row =\( ((1.34, 1.43, 1.52); 0.50 ,0.50, 0.50) \),
   The weighted sum of the second row =\( ((1.38, 1.47, 1.56); 0.50 ,0.50, 0.50) \),
   The weighted sum of the third row =\( ((0.23, 0.28, 0.35); 0.30 ,0.75, 0.70) \),
   The weighted sum of the fourth row =\( ((0.21, 0.26, 0.30); 0.30 ,0.75, 0.70) \),
   The weighted sum of the fifth row =\( ((0.32, 0.34, 0.46); 0.30 ,0.75, 0.70) \).
2. By dividing the values of the weighted sum vector by the corresponding priority for each criterion we obtain the following :
   \[
   \frac{((1.34, 1.43, 1.52); 0.50 ,0.50, 0.50)}{0.4}, \quad \frac{((1.38, 1.47, 1.56); 0.50 ,0.50, 0.50)}{0.5}, \quad \frac{((0.23, 0.28, 0.35); 0.30 ,0.75, 0.70)}{0.03}, \quad \frac{((0.21, 0.26, 0.30); 0.30 ,0.75, 0.70)}{0.04}, \quad \frac{((0.32, 0.34, 0.46); 0.30 ,0.75, 0.70)}{0.02}.
   \]
3. Compute the mean of the values found in the previous step; this mean is denoted \( \bar{\lambda}_{max} \), then
   \[
   \bar{\lambda}_{max} = \text{Average}\{\frac{((1.34, 1.43, 1.52); 0.50 ,0.50, 0.50)}{0.4}, \quad \frac{((1.38, 1.47, 1.56); 0.50 ,0.50, 0.50)}{0.5}, \quad \frac{((0.23, 0.28, 0.35); 0.30 ,0.75, 0.70)}{0.03}, \quad \frac{((0.21, 0.26, 0.30); 0.30 ,0.75, 0.70)}{0.04}, \quad \frac{((0.32, 0.34, 0.46); 0.30 ,0.75, 0.70)}{0.02}\}\}
   = \{(7,8,9); 0.30 ,0.75, 0.70\}.
   
   Since \( \bar{\lambda}_{max} \) still neutrosophic number, then we need to de-neutrosophic it as follows:
   \[
   S(\bar{\lambda}_{max}) = \left| \bar{\lambda}_{max} + m_{\bar{\lambda}_{max}} + u_{\bar{\lambda}_{max}} \right| / 3 + (T_{\bar{\lambda}_{max}} - I_{\bar{\lambda}_{max}} - F_{\bar{\lambda}_{max}}),
   \]
   Then \( \lambda_{max} = 6.85 \).
4. Compute the consistency index (CI) as follows:

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1} \frac{6.85 - 5}{4} = 0.46.
\]

Where \( n \) is the number of criteria being compared.

5. Compute the consistency ratio, which is defined as:

\[
\text{CR} = \frac{CI}{RI} = \frac{0.46}{1.12} = 0.4
\]

Since the comparison matrix is \( 5 \times 5 \) (i.e. \( n=5 \)), the CR should be \( \leq 1.12 \) as presented in Table 4, so it’s a very compatible ratio of the comparison matrix, but we can also improve this ratio and make it very near to 0.1 to increase degree of consistency during the following steps:

1. Construct the neutrosophic induced matrix \( \tilde{I} = \tilde{A} \cdot \tilde{A} - n \cdot \tilde{A} \).

\[
\tilde{I} = \begin{bmatrix}
(0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (5.13, 28): 0.30, 0.75, 0.70 & (5.13, 23): 0.30, 0.75, 0.70 & (3.8, 16): 0.30, 0.75, 0.70 \\
(0.50, 0.50, 0.50) & (5.13, 28): 0.30, 0.75, 0.70 & (0.50, 0.50, 0.50) & (5.13, 23): 0.30, 0.75, 0.70 & (3.8, 16): 0.30, 0.75, 0.70 \\
(0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (5.13, 23): 0.30, 0.75, 0.70 & (5.13, 28): 0.30, 0.75, 0.70 & (0.50, 0.50, 0.50) \\
(0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (5.13, 23): 0.30, 0.75, 0.70 & (5.13, 28): 0.30, 0.75, 0.70 \\
(0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (0.50, 0.50, 0.50)
\end{bmatrix}
\]

2. The largest preference relation \( \tilde{r}_{ij} \), which has the largest lower, median and upper-bound of triangular number is \( \tilde{r}_{24} \).

3. The dot product

\[
\tilde{P} = \tilde{R}_{O_2} \cdot \tilde{C}_{O_2} = \{(5, 0, 23): 0.30, 0.75, 0.70 \} \}
\]

4. Calculate elements which are distant from \( \tilde{r}_{ij} \) in vector \( \tilde{P} \) by the following formula:

\[
\tilde{B} = \tilde{P} - \tilde{r}_{24} = \{(0.30, 0.75, 0.70)\} \}
\]

5. All elements with zero or negative values for each lower, median and upper bound of triangular number in \( \tilde{B} \) is consistent element and we should enhance other elements in \( \tilde{B} \).

To improve consistency of original pair-wise comparison matrix, we try to modify \( \tilde{r}_{24} \) and \( \tilde{r}_{25} \) as Table 8.

**Table 8.** The modified comparison matrix of criteria
We found that it's an efficient ratio by comparing the consistency ratio CR from 0.4 to 0.17 and it's an efficient ratio by comparing it with 1.12 as in Table 4 of Saaty.

The priorities of criteria are presented in Fig. 5 as follows:

\[ C_1, C_2, C_4, C_3 \] and \( C_5 \) respectively and this means that, security and performance are the most important criteria according to company's directors.

By calculating \( \lambda_{max} \) as we illustrated previously with details, we found that \( \lambda_{max} = 5.85 \).

Compute the consistency index (CI) as follows:

\[ CI = \frac{\lambda_{max} - n}{n-1} \frac{5.85 - 5}{4} = 0.2 \]

\[ CR = \frac{CI}{RI} \frac{0.2}{1.12} = 0.17 \]

It's obvious that CR became close to 0.1, and we reduced the consistency rate CR from 0.4 to 0.17 and it's an efficient ratio by comparing it with 1.12 as in Table 4 of Saaty.

**Figure. 5.** The weight obtained for the evaluation criteria

To estimate the benefit and applicability of the previous proposed criteria in this research, we focus on the four criteria which are determined from IEEE Standard [65]:

This standard is a method to establish quality requirements and identify, implement,
analyze, and validate any process or product. Then by applying this standard on proposed criteria, we ensure that the determined criteria are valid and establish quality requirements. Then, the selected product (cloud service provider) will be the best and a high quality product. The four criteria which are determined from IEEE Standard are as follows:

- **Correlation**: in order to show the interdependency between criteria, we will use the correlation coefficient of Spearman through the following formula:
  \[
  \rho = 1 - \frac{6 \sum D_i^2}{n(n^2 - 1)} \tag{16}
  \]
  Where \(\rho\) is the correlation coefficient, \(D_i = x_i - y_i\) and it's the difference between ranked criteria's values and \(n\) is the number of criteria.
  
  \(\rho_{\text{security-performance}} = 0.99\) and this means that the correlation between security and performance is very high. Also \(\rho_{\text{security-accessibility}} = 0.77\), \(\rho_{\text{security-stability}} = 0.78\), \(\rho_{\text{security-adaptability}} = 0.77\), \(\rho_{\text{performance-accessibility}} = 0.86\), \(\rho_{\text{performance-stability}} = 0.87\), \(\rho_{\text{performance-adaptability}} = 0.86\), \(\rho_{\text{accessibility-stability}} = 0.99\), \(\rho_{\text{accessibility-adaptability}} = 1\), \(\rho_{\text{stability-adaptability}} = 0.99\). It's obvious that the correlation between criteria is very high.

- **Consistency of criteria**: as we illustrated in the previous by calculating consistency ratio of criteria, it's noted that the criteria are consistent.

- **Computability and practicability of criteria**: it's obvious that the proposed criteria are easy to compute and practical.

- **Power of discriminative**: the proposed criteria can handle various cloud providers.

**Step 5**: Determine weights of alternatives according to each criterion.

The pair-wise comparison matrix of alternatives according to security criterion is presented in table 9.

**Table 9.** The pair-wise comparison matrix of alternatives according to security
A1, A2 and A3 are Dropbox, Google Drive and Microsoft Sky Drive respectively.

Similar to weight calculation methodology of criteria, we will also calculate the normalized weight of alternatives as follows:

1. Take the totality row averages using the following equation:

\[ \tilde{w}_i = \frac{\sum_{j=1}^{n} (t_{ij} m_{ij}, u_{ij}) T_{ij}, T_{ij}, F_{ij})}{n} \]

Then,

\[ \tilde{w}_1 = \langle (0.4, 0.44, 0.45); 0.50, 0.50, 0.50 \rangle, \]
\[ \tilde{w}_2 = \langle (1.75, 2.1, 2.5); 0.30, 0.75, 0.70 \rangle, \]
\[ \tilde{w}_3 = \langle (4.4, 3, 4.7); 0.30, 0.75, 0.70 \rangle. \]

2. Since \( \tilde{w}_i \) are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq.9.

\[ w_1 = 0.07, w_2 = 0.96, w_3 = 3.2 \]

3. After de-neutrosophic \( \tilde{w}_i \), we obtain \( w_i \), which is a crisp value and we need to normalize it using Eq.10. then,

\[ w_1 = 0.4, w_2 = 0.5, w_3 = 0.03 \]

Then, the weight of Dropbox according to security criterion is 0.4, weight of Google Drive according to security criterion is 0.5 and weight of Microsoft Sky Drive according to security criterion is 0.03. These values are basically dependent on decision makers comparison matrix, according to their opinions and requirements of e-learning company.

The pair-wise comparison matrix of alternatives according to performance criterion is presented in Table 10.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>\langle (1, 1); 0.50, 0.50, 0.50 \rangle</td>
<td>\langle (1 1, 1); 0.80, 0.15, 0.20 \rangle</td>
<td>\langle (1 1, 1); 0.90, 0.10, 0.10 \rangle</td>
</tr>
<tr>
<td>A2</td>
<td>\langle (4, 5); 0.80, 0.15, 0.20 \rangle</td>
<td>\langle (1, 1); 0.50, 0.50, 0.50 \rangle</td>
<td>\langle (1 1, 1); 0.80, 0.15, 0.20 \rangle</td>
</tr>
<tr>
<td>A3</td>
<td>\langle (6, 7); 0.90, 0.10, 0.10 \rangle</td>
<td>\langle (4, 5); 0.80, 0.15, 0.20 \rangle</td>
<td>\langle (1, 1); 0.50, 0.50, 0.50 \rangle</td>
</tr>
</tbody>
</table>
The normalized weight of alternatives according to performance criterion is as follows:

1. Take the totality row averages using the following equation:
   \[ \tilde{w}_i = \frac{\sum_{j=1}^{n}(l_{ij}m_{ij}u_{ij};T_{ij},l_{ij}F_{ij})}{n} \]
   Then, \[ \tilde{w}_1 = \langle (0.43, 0.44, 0.47); 0.50, 0.50, 0.50 \rangle, \]
   \[ \tilde{w}_2 = \langle (1.7, 2.2, 5); 0.50, 0.50, 0.50 \rangle, \]
   \[ \tilde{w}_3 = \langle (3.7, 4.3, 5); 0.50, 0.50, 0.50 \rangle. \]

2. Since \( \tilde{w}_i \) are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq.9.
   \( w_1 = 0.05, w_2 = 1.5, w_3 = 3.8 \).

3. After de-neutrosophic \( \tilde{w}_i \), we obtain \( w_i \), which is a crisp value and we need to normalize it using Eq.10, Then
   \( w_1 = 0.01, w_2 = 0.3, w_3 = 0.7 \).

Then, the weight of Dropbox according to performance criterion is 0.01, weight of Google Drive, according to performance criterion is 0.3 and weight of Microsoft Sky Drive according to performance criterion is 0.7. These values are basically depend on decision makers comparison matrix, according to their opinions and requirements of e-learning company.

The pair-wise comparison matrix of alternatives according to accessibility criterion is presented in table 11.

**Table 11.** The pair-wise comparison matrix of alternatives according to accessibility

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Alternative A1</th>
<th>Alternative A2</th>
<th>Alternative A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>( (1,1); 0.5, 0.5, 0.5 )</td>
<td>( (6,7); 0.9, 0.1, 0.1 )</td>
<td>( (\frac{1}{6}, \frac{1}{5}, 0.4); 0.8, 0.15, 0.2 )</td>
</tr>
<tr>
<td>A2</td>
<td>( (1,1); 1, 1, \frac{7}{6} )</td>
<td>( (1,1); 0.5, 0.5, 0.5 )</td>
<td>( (9, 9); 1.0, 0.0, 0.0 )</td>
</tr>
<tr>
<td>A3</td>
<td>( (4,5,6); 0.8, 0.1, 0.2 )</td>
<td>( (\frac{1}{5}, \frac{1}{5}, 0.7); 1.0, 0.0, 0.0 )</td>
<td>( (1,1,1); 0.5, 0.5, 0.5 )</td>
</tr>
</tbody>
</table>

The normalized weight of alternatives with according to accessibility criterion is as follows:

1. Take the totality row averages using the following equation:
   \[ \tilde{w}_i = \frac{\sum_{j=1}^{n}(l_{ij}m_{ij}u_{ij};T_{ij},l_{ij}F_{ij})}{n} \]
   Then, \[ \tilde{w}_1 = \langle (2.4, 2.7, 3); 0.5, 0.5, 0.5 \rangle, \]
\[ \hat{w}_2 = (3.4, 3.4, 3.4); 0.50, 0.50, 0.50 \]
\[ \hat{w}_3 = (1.7, 2, 2.4); 0.50, 0.50, 0.50. \]

2. Since \( \hat{w}_i \) are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq.9.
\[ w_1 = 2.2, w_2 = 2.9, w_3 = 1.5. \]

3. After de-neutrosophic \( \hat{w}_i \), we obtain \( w_i \), which is a crisp value and we need to normalize it using Eq.10. Then,
\[ w_1 = 0.33, w_2 = 0.44, w_3 = 0.23 \]

Then, the weight of Dropbox according to accessibility criterion is 0.33, weight of Google Drive, according to accessibility criterion is 0.44 and weight of Microsoft Sky Drive according to accessibility criterion is 0.23. These values are basically dependent on decision makers comparison matrix, according to their opinions and requirements of e-learning company.

The pair-wise comparison matrix of alternatives according to scalability criterion is presented in table 12.

**Table 12.** The pair-wise comparison matrix of alternatives according to scalability

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A₁</th>
<th>A₂</th>
<th>A₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>(1,1,1); 0.50,0.50,0.50</td>
<td>(9,9,9); 1.00,0.00,0.00</td>
<td>(6,7,8); 0.90,0.10,0.10</td>
</tr>
<tr>
<td>A₂</td>
<td>(1,1,1); 0.50,0.50,0.50</td>
<td>(9,9,9); 1.00,0.00,0.00</td>
<td>(1,1,1); 0.50,0.50,0.50</td>
</tr>
<tr>
<td>A₃</td>
<td>(9,9,9); 1.00,0.00,0.00</td>
<td>(1,1,1); 0.50,0.50,0.50</td>
<td>(1,1,1); 0.50,0.50,0.50</td>
</tr>
</tbody>
</table>

The normalized weight of alternatives with according to scalability criterion is as follows:

1. Take the totality row averages using the following equation:
\[ \tilde{w}_i = \frac{\sum_{j=1}^n (l_{ij} m_{ij} u_{ij}) T_{ij} l_{ij} F_{ij}}{n}, \]

Then,
\[ \tilde{w}_1 = (5.3, 5.7, 6); 0.50, 0.50, 0.50, \]
\[ \tilde{w}_2 = (3.4, 3.4, 3.4); 0.50, 0.50, 0.50, \]
\[ \tilde{w}_3 = (0.4, 0.42, 0.4); 0.50, 0.50, 0.50. \]

2. Since \( \tilde{w}_i \) are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq.9.
\[ w_1 = 5.2, w_2 = 2.9, w_3 = 0.9. \]

3. After de-neutrosophic \( \tilde{w}_i \), we obtain \( w_i \), which is a crisp value and we need to normalize it using Eq.10. Then,

\[
w_1 = 0.63, w_2 = 0.35, w_3 = 0.01
\]

Then, the weight of Dropbox according to scalability criterion is 0.63, weight of Google Drive according to scalability criterion is 0.35 and weight of Microsoft Sky Drive according to scalability criterion is 0.01.

The pair-wise comparison matrix of alternatives according to adaptability criterion is presented in Table 13.

**Table 13.** The pair-wise comparison matrix of alternatives according to adaptability

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A₁</th>
<th>A₂</th>
<th>A₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>((1,1,1);0.50,0.50,0.50)</td>
<td>((\frac{1}{5},\frac{1}{5},\frac{1}{5});0.90,0.10,0.10)</td>
<td>((\frac{1}{5},\frac{1}{5},\frac{1}{5});1.00,0.00,0.00)</td>
</tr>
<tr>
<td>A₂</td>
<td>((6,7,8);0.90,0.10,0.10)</td>
<td>((1,1,1);0.50,0.50,0.50)</td>
<td>((2,3,4);0.30,0.75,0.70)</td>
</tr>
<tr>
<td>A₃</td>
<td>((9,9,9);1.00,0.00,0.00)</td>
<td>((\frac{1}{5},\frac{1}{5},\frac{1}{5});0.30,0.75,0.70)</td>
<td>((1,1,1);0.50,0.50,0.50)</td>
</tr>
</tbody>
</table>

The normalized weight of alternatives with respect to adaptability criterion is as follows:

1. Take the totality row averages using the following equation:

\[
\tilde{w}_i = \frac{\sum_{j=1}^{n}(l_{ij},m_{ij},u_{ij}) \cdot T_{ij} \cdot J_{ij} \cdot F_{ij}}{n}
\]

Then,

\[ \tilde{w}_1 = \langle (0.4, 0.42, 0.42); 0.50, 0.50, 0.50 \rangle, \]
\[ \tilde{w}_2 = \langle (3, 3.6, 4.3); 0.30, 0.75, 0.7 \rangle, \]
\[ \tilde{w}_3 = \langle (3.4, 3.4, 3.5); 0.30, 0.75, 0.7 \rangle. \]

2. Since \( \tilde{w}_i \) are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq.9.

\[ w_1 = 0.09, w_2 = 2.5, w_3 = 2.3. \]

3. After de-neutrosophic \( \tilde{w}_i \), we obtain \( w \), which is a crisp value and we need to normalize it using Eq.10, then:

\[
w_1 = 0.02, w_2 = 0.51, w_3 = 0.47
\]

The weight of three clouds computing according to each criterion presented in Fig.6.
Figure 6. Comparison of three cloud computing according to different criteria

Step 6: Calculate the scores of alternatives by multiplying the weight of each alternative with the related criteria.

Then the relative scores for each alternative as follows:
Step 7: Rank alternatives according to the largest score value.

Since the weight of Dropbox is 0.24, weight of Google Drive is 0.41 and weight of Microsoft Sky Drive is 0.32, by comparing them from decision makers, according to proposed criteria and e-learning company requirements. Then, the rank of alternatives is as follows: Google Drive followed by Microsoft Sky Drive and Dropbox as in Fig.7. Since Google Drive has the highest weight comparing to two other drives with respect to predefined criteria and e-learning company requirements. Then, we recommended to the e-learning company the selection of Google Drive, because it is the better choice taking into account all the determined criteria and the preference of decision makers judgment.

![Figure 7. The priorities of alternatives](image)

6. Related Work and Model Evaluation

In this section, the neutrosophic multi-criteria decision analysis methodology depends on the analytic hierarchy process for estimating the quality of cloud services has been evaluated and compared with other existent methods:

The analytic hierarchy process has been used by Garg et al.[8] to estimate the effectiveness of cloud services in an organization and this method has some drawbacks such as: The failure to handle vague, inconsistent information and the altitude cognitive request on the decision makers. They also did not provide any method to repairs the inconsistency of judgments.

The fuzzy AHP has been used by Safari et al.[55] for prioritizing cloud computing acceptance indicators, by Singla and Kaushal to allow users to select an optimal cloud
service[56], by Cheng [57] for cloud computing decision making problem and a fuzzy multi-criteria group decision making method based on the technique for order preference by similarity to the ideal solution (TOPSIS) has been used by Wibowo et al [60] for evaluating cloud services. But these methods have some drawbacks such as: It should not represent real life situation efficiently, because it considers only the membership function and didn't take into account the indeterminacy and falsity function. Also, the scale used in Fuzzy HP cannot reflect the perceptions of the decision maker accurately. They didn't provide any technique to repair the judgments and make it consistent.

We overcame the previous drawbacks by proposing a simple model to evaluate the effectiveness of cloud services and select the optimal choice. Our proposed model provides the user with a richer structure framework than the classical and fuzzy AHP. Our model can handle vagueness and uncertainty over classical AHP and fuzzy AHP because it considers three different grades "membership degree, indeterminacy degree and non-membership degree". We also pointed out how to repair inconsistent judgments by developing the induced bias matrix and applying it in neutrosophic surroundings.

By applying the eight quality factors, which were proposed by Moody and Shanks[66] to estimate the quality of our proposed model we found that:

1. The main criteria to estimate cloud services are presented, so our proposal is complete.
2. The decision maker can add or remove criteria for adjusting the proposed model to his/her organization, so our proposal is flexible.
3. The proposal is easy to understand.
4. The proposal is valid and correct.
5. Our proposal is simple to apply as we verified with example.
6. Our proposal is integrity.
7. Our proposal has an implement-ability feature.
8. Our proposal can help organization to make the best choice because its consistent with problem.
7. Conclusions and Future Works

Clouds are computing and data-storage systems which cement different technologies to interconnect and control demanded resources which are on dispensed computers. Because of the fast development of cloud computing, many cloud services have appeared. Any organization tries to achieve best flexibility and quick response to market requests, and for these reasons it is trying to use cloud services. Due to the variety of cloud service providers, it is a very significant defy for organizations to select the appropriate cloud services which can fulfill their needs. This research has introduced a neutrosophic multi-criteria decision analysis method depending on the analytic hierarchy process for estimating the performance of cloud services. A group of decision makers consulted to compare alternatives according to various criteria. The preferences of decision makers are aggregated to achieve consensus of decision makers. The consistency degree of pairwise comparison matrix calculated in this research not only this, but we also improved consistency degree via representing induced bias matrix in neutrosophic environment. The estimation process of cloud services is modeled by using triangular neutrosophic numbers represented by linguistic variables in comparison matrices. A score function is introduced to transform triangular neutrosophic number to its equivalent crisp value. A real example of a firm in Egypt, is solved to check applicability of proposed technique. The suggested neutrosophic multi-criteria decision analysis approach has achieved many benefits for transacting with ambiguous and inconsistent information which exist usually in cloud services estimation process. In the future, we will estimate the performance of cloud computing services by using different multi-criteria decision analysis approach and compare between their results.

Limitation of Proposed Research: More involvements from more companies will make our research better.

Competing Interests

The authors announce that there is no discrepancy of interests concerning the publication of this research.
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